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A REVIEW OF THE HISTORY OF JAPANESE MATHEMATICS

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ABSTRACT. — This review aims to introduce Japanese mathematics to a non-expert and a non-Japanese readership. It briefly characterizes mathematics in Japan, surveys its history, as it developed over the last century, and provides a large (if not exhaustive) bibliography of works in the primary European languages.

RÉSUMÉ. — APERÇU SUR L'HISTOIRE DES MATHÉMATIQUES JAPONAISES. Le but de cette note est de présenter les mathématiques japonaises à un public non spécialisé dans le domaine. Les mathématiques au Japon sont brièvement caractérisées, leur histoire, telle qu'elle s'est développée durant le dernier siècle, est passée en revue et finalement une importante bibliographie dans les principales langues européennes est proposée, même si elle ne peut prétendre à l'exhaustivité.

1. INTRODUCTION – NOT ONLY *SANGAKU*

The custom of hanging *sangaku* (算額), wooden plates on which are inscribed mathematical problems and their answers, under the roofs of shrines in the Edo period (1603–1867) in Japan is familiar enough to have been illustrated and described in *Scientific American*, May 1998 [Rothman 1998]. It is, however, obvious that sangaku is not all there is to Japanese mathematics. It would be fallacious to consider that the essence of Japanese mathematics reveals itself in sangaku.

As a beginning, this essay attempts briefly to introduce Japanese mathematics to a non-Japanese readership. It will answer the following questions :

1) What kind of mathematical disciplines developed in Japan ?
2) How were mathematical expressions written in vertical typesetting ?
3) How did the Japanese calculate ?
4) Is it true that, lacking proofs, Japanese mathematics was not logical ?

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5) Is Japanese mathematics just a poor imitation of Chinese mathematics?

6) Was Japanese mathematics useful in other fields?

7) How was mathematics learned in Japan?

It will then look back on 100 years of study of the history of Japanese mathematics, dividing it into six areas of study. The survey will be limited mainly to treatises written in European languages, and will close with a bibliography of articles written in European languages on the history of Japanese mathematics. Although it is not exhaustive, I hope it will be useful for all readers interested in Japanese mathematics.

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2. PECULIARITY OF JAPANESE MATHEMATICS

Japanese mathematics (*Wasan*, 和算) is defined as the mathematics developed in Japan before the Meiji Restoration in the latter half of the 19th century when Japan was forced to end its seclusion and was exposed to Western culture.

It flourished especially in the Edo period when Japan was a closed country[1]. This means that it was one of the last non-European mathematical traditions to westernize.

2.1. What kind of mathematical disciplines developed in Japan?

Various domains are represented in Japanese mathematics. Articles in the bibliography, for example, concern the plane geometry of polygons, circles, ellipses, number theory of indefinite equations and Pythagorean triangles, theory of determinants, problems concerning sums of progressions, and so on[2]. There are other disciplines represented such as solid

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[1] [Hayashi 1903–1905, 1907b], [Mikami 1974], [Ogura 1993], and [Smith and Mikami, 1914] are complete histories. Although [Smith and Mikami 1914] is standard, [Ogura 1993] is also suitable as a primer. It was first published in 1940 and has only 100 pages, but it is illustrated and is easy to understand. The fact that these works first appeared 50 to 100 of years ago suggests that the historical study of Japanese mathematics has developed rather slowly during the last fifty years. However, the situation has changed considerably since the 1990's. The time is now ripe for a new complete history of Japanese mathematics.

[2] On plane geometry, see [Mikami 1915] and [Shinomiya and Hayashi 1917]. See,

geometry. One of the most brilliant achievements among these is *enri* (円理), or the circle principle, the general term for analytical methods of calculating lengths of circles, arcs, and other curves, or of computing volumes or surface areas of solids[3]. For example, Takebe Katahiro (建部賢弘, 1664–1739) calculated the value of π to 41 decimal places in 1722 using a numerical method similar to Richardson's extrapolation method[4] and also got an infinite series,

$$\left(\frac{s}{2}\right)^2 = cd\left\{1 + \frac{2^2}{3\cdot 4}\left(\frac{c}{d}\right) + \frac{2^2\cdot 4^2}{3\cdot 4\cdot 5\cdot 6}\left(\frac{c}{d}\right)^2 + \frac{2^2\cdot 4^2\cdot 6^2}{3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}\left(\frac{c}{d}\right)^3 + \cdots\right\},$$

where d is the diameter of the given circle, c and s are the sagitta and the length of an arc respectively[5].

2.2. How were mathematical expressions written in vertical typesetting?

Today, the Japanese write sentences both horizontally and vertically, but in the Edo period, they only wrote vertically. Mathematics was also written vertically in Japan (as it was in China). But there were some improvements in Japanese mathematics. They were accomplished by Seki Takakazu (関孝和, ?–1708)[6] and his successor, Matsunaga Yoshisuke (松永良弼, ?–1744), in the second half of the 17th century and called *Tenzan Jutsu* (點竄術). They introduced characters and frac-

in particular, [Mikami 1912b] and [Yanagihara 1915] for ellipses. For a discussion of the solutions to the indefinite equation $x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2 = y^2$ by Ajima Naonobu (安島直円, 1732–1798) and Pythagorean triangles, see [Mikami 1912a] and [Yanagihara 1914–1915], respectively. [Hayashi 1909–1910] and [Mikami 1909–1910b, 1914–1919] deal with the theory of determinants and [Yanagihara 1918] analyzes various progressions that appear in Japanese mathematics.

3 See [Harzer 1905] and [Mikami 1909–1910a, 1930] for an outline. For further details of enri before Takebe Katahiro, see [Horiuchi 1994a, 1994b]. [Mikami 1914b] analyzes the representation of π by an infinite product in *Suuri Shinpen* (数理神篇, 1860) by Saitou Noriyoshi (斎藤宜義, 1816–1889). See also [Mikami 1913a,b,c,d], [Mikami 1914a,b], and [Nakamura 1994]. See [Horiuchi 1994a] for a recent account in French.

4 See [Horiuchi 1994a] or [Xu 1999] for more details. Takebe said in his treatise *Tetsujutsu sankei* (綴術算経, 1722) that he had calculated π^2. Both Horiuchi and Xu explain this calculation, but the values Takebe wrote down in it are, in fact, for calculating π itself.

5 See [Kikuchi 1896] and [Horiuchi 1994a,b].

6 Seki is one of the most famous and brilliant mathematicians in the history of Japanese mathematics, and many authors mention him. [Hayashi 1906b,c], [Jochi 1993], [Kikuchi 1896], [Mikami 1908], and [Ogawa 1995] contain the word 'Seki' in the title directly.

tional expressions. Matsunaga, for example, wrote expressions $ab/h + h$ and $a^2b^2/h^2 + 2ab + h^2$ as (a) and (b) in the next figure respectively.[7] These would be as (a') and (b') if we replace Chinese characters with alphabetic representations.

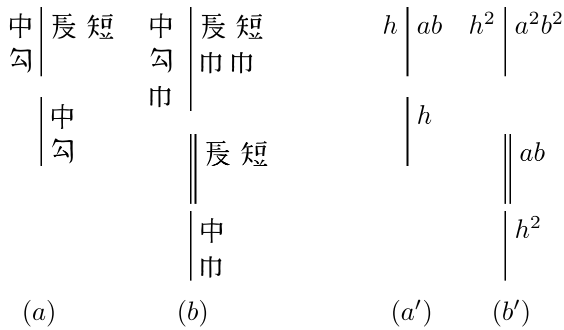

Figure 1. *Expressions in Japanese mathematics.*

In some cases, these formulae mean simple and quadratic equations $ab/h + hx = 0$ and $a^2b^2/h^2 + 2abx + h^2x^2 = 0$ in the unknown x, but no confusion arises in context.[8] *Tenzan Jutsu* had the potential to express any complicated calculation, but, since there were few symbols other than the ones in figure 1, they had to complement calculation with sentences. Takebe's formula in 2.1 was also described in this manner. This style was widely used until the end of Japanese mathematics. Today, it is somewhat difficult to follow such calculations, especially when they involve many unknowns.

2.3. How did the Japanese calculate?

Japanese mathematicians mainly used abaci for numerical computation. Abaci were tools imported from China that spread throughout the country. Although it is uncertain when they were imported into Japan, the Japanese used them for calculation in both private and business settings until the advent of the electric calculator in the latter half of the

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[7] *Enchuu San'gen Tekitou* (円中三原適等). Some Chinese characters are omitted from the expressions.

[8] See [Hayashi 1905 p. 340–346] for further details.

20th century[9]. The abacus was imported from China together with books such as *Suanfa Tongzong* (算法統宗, Cheng Dawei in the Ming Dynasty, 1593). People in the Edo period could perform not only addition, subtraction, multiplication, and division but also square root extraction with the abacus. Moreover, they solved equations of any degree in one variable using multiple abaci simultaneously. This method had its origin in a Chinese method using computing rods.[10] Since the Japanese used the abacus, for calculation, they seldom wrote down the process of calculation.

Is it true that, lacking proofs, Japanese mathematics was not logical?

It is often said that Japanese mathematics was not constructed logically and that it contained no proofs. When a Chinese translation of Euclid's *Elements* by an Italian missionary in China, Matteo Ricci (1552–1610) who collaborated with Xu Guangqi (徐光啓), was introduced into Japan, Japanese mathematicians did not recognize that it aimed at building the edifice of mathematical knowledge[11]. It is certain that there were few definitions, postulates, and even proofs in Japanese mathematics. But this does not imply that Japanese mathematics represented a collection of illogical and unfounded assertions. People in the Edo period familiarized themselves with the famous theorem of Pythagoras and understood many kinds of proofs of it, though they were not interested in fundamental concepts like points, lines, and so on. After all, mathematics was regarded not as a deductive study, but as an inductive one. In the case of the calculation of π or the length of an arc, mathematicians in the Edo period investigated empirically and drew what seemed to be irrefutable conclu-

[9] Today, few in Japan use abaci regularly, but there are still calculation contests in which the contestants manipulate abaci with great speed. They can answer 10 questions of adding or subtracting 15 numbers with 10 digits within 4 minutes, and 20 questions of multiplying or dividing two numbers with 6 digits within 3 minutes. Some of these virtuosi never actually handle the abacus, since they calculate by visualizing the abacus' movements. It is astonishing that they can calculate the next question as they write down the answer of the previous question. Their special skills are certainly superior to those of the people in the Edo period, but Takebe might have been able to calculate faster and more accurately than we expect.

[10] Hayashi [1903–1905, p. 313–324], explains how to use computing rods (*sangi* (算木), in Japanese). See also [Mikami 1911c].

[11] Ricci's *Elements*, that was translated from Clavius's, contained only the first six books. They were introduced into Japan as a part of *Tianxue Chuhan* (天学初函).