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FLIPS AND ABUNDANCE FOR ALGEBRAIC THREEFOLDS

János KOLLÁR

A summer seminar at the University of Utah
(Salt Lake City, 1991)

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PREFACE

These notes originated at the Second Algebraic Geometry Summer Seminar held at the University of Utah during August 1991. The seminar was the continuation of the First Summer Seminar held in 1987 whose notes appeared in [CKM88].

The aim of the First Summer Seminar was to give an introduction to three dimensional birational geometry, especially to Mori's Program (also called the Minimal Model Program). We are very happy to note that in the last few years this program has become much better known among algebraic geometers. This was reflected in the number of participants. In 1987 there were 16 participants for an introductory seminar; in 1991 there were 30 for a more advanced one.

Because of these changes, instead of starting at the beginning, the Second Summer Seminar concentrated on reviewing recent developments in higher dimensional birational geometry. We surveyed two of the most important recent directions.

The first topic was the existence of flips in dimension three, the final step in the three dimensional Minimal Model Program. In surface theory it is well known that repeated contraction of -1-curves yields a minimal surface. Similarly, starting with a threefold X, Mori's Program produces another threefold X', birational to X, which can reasonably be called minimal in analogy with the surface case. The required operations are however more complicated. One of them is called flip.

The existence of flips was first proved by [Mori88]. Recently a very different approach to a more general type of flipping problem (still in dimension three) was found by [Shokurov91]. We owe special thanks to Miles Reid who prepared an English translation of [Shokurov91] in a very short time. Shokurov's article contains many new ideas, but unfortunately it is very difficult to understand. Numerous parts required a truly joint effort of the participants and some details were understood only after several discussions with the author. Eventually we discovered an error in [ibid, 8.3]. Unfortunately, there was no opportunity to reconvene the seminar and study the new version [Shokurov92].

The first part of the notes (Chapters 4–8) presents a new proof of log flips using [Mori88]. The third part (Chapters 16–21) presents a reworked version of [Shokurov91, 1–7].

The second topic (Chapters 9–15) is the Abundance Conjecture proposed in [Reid83]. It is a natural continuation of Mori's Program. Starting with the threefold X' produced above, the conjecture states that a suitable multiple of the canonical class determines a base point free linear system (unless all such are empty). The proof of this result was completed in the series of articles [Kawamata84,85,91b; Miyaoka87a,b,88a,b]. Again we succeeded in simplifying several of the steps and generalizing many intermediate results.

A more detailed explanation of the results and an outline of the proofs is given in Chapter 1.

ACKNOWLEDGEMENT. We are very grateful to S. Mori for his attention and help during and after the conference. He pointed out several mistakes in preliminary versions of the notes.

Many errors and inaccuracies were pointed out to us by S. Kovács and E. Szabó. We received long lists of comments, corrections and improvements from M. Reid and from V. V. Shokurov. They helped to improve these notes considerably.

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CONTENTS

LOG FLIPS II

PREREQUISITES

In writing these notes we tried to keep the prerequisites to the minimum. The reader is assumed to have a basic general knowledge of algebraic geometry. Some familiarity with higher dimensional techniques is necessary. We tried to rely only on Chapters 1–13 of [CKM88]. There are however two topics not adequately covered in [CKM88].

1. In [CKM88] the Cone Theorem and related results are proved only for the canonical divisor K_X instead of an arbitrary log terminal divisor $K_X + \Delta$. The proofs in the more general log terminal case are essentially the same as the proofs given in [CKM88]. A reader who understands Chapters 9–13 of [CKM88] should have no problem with the more general log versions. However we usually refer to [KMM87] where the precise results are stated and proved.

2. [CKM88, Chapter 6] collects the most important results on terminal and canonical singularities in dimension three, mostly without proofs. The reader who is happy to accept these results does not need to know more. For those who want proofs, the list of prerequisites gets longer. The survey article [Reid87] presents a very readable and elementary overview with proofs. Unfortunately even [Reid87] relies on detailed properties of elliptic Gorenstein surface singularities [Laufer77; Reid75] which are by no means basic. We could not offer any significant improvements; thus there was no reason to reproduce the results.

3. The first proof of the existence of log flips (Chapters 4–8) uses the very difficult results of [Mori88]. We need however only the statements and none of the techniques.

4. In Chapter 9 we discuss the abundance problem only for regular threefolds. The irregular case was solved earlier using the ideas of Iitaka's program which are not related to the methods discussed here.

No other result from higher dimensional birational geometry is used without proof.

We also need some other results which are not part of basic algebraic geometry.

Naturally we need Hironaka's resolution of singularities.

Simultaneous resolution of flat deformations of Du Val singularities (= rational double points) is an important result [Brieskorn71] which is not treated in textbooks.

In Chapters 9–10 we use several properties of stable vector bundles. Also in Chapter 9 we need some properties of foliations in positive characteristic. In all cases we state the results we use and give precise references.

Finally there are occasional uses of a few other topics: mixed Hodge structures, Lefschetz type theorems, relative duality and the existence of the Hilbert scheme.

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