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NOTES & DÉBATS

DEUTERONOMIC TEXTS: LATE ANTIQUITY AND THE HISTORY OF MATHEMATICS [1]

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Introduction

In this article I offer a reassessment of Late Antiquity and the Middle Ages in the history of mathematics. For this purpose, I develop a more general notion, of "deuteronomic" texts, *i.e.* texts depending fundamentally on earlier texts. I describe in detail some of the features typical to this period in the history of Western mathematics — Late Antiquity and the Middle Ages — where deuteronomic texts were crucial. I then argue briefly that those features had significant consequences in a changing practice, and image, of mathematics, and also that those features derive directly from the role of deuteronomic texts. Thus the argument is that Late Antiquity and the Middle Ages had a real historical contribution to make; and that this derived from the basic nature of texts produced in this period.

Now Late Antiquity — and, largely speaking, the Middle Ages — did not fare well with the historians of mathematics. Pappus[2] — to take the

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[1] This article owes its inception to a presentation, followed by an especially vigorous discussion, in the QED conference at the Max Planck Institute for the History of Science, Berlin, May 1998. I wish to thank the convenor of the conference, Loraine Daston, and all of the discussants. I also wish to thank Karine Chemla, Catherine Goldstein and Alain Herreman for their inspiration in the preparation of this article.

[2] Active in 4th century AD Alexandria, his biography is practically unknown. Dealing with a wide range of topics from arithmetic to mechanics, his most significant work is *The Collection*, a sort of mathematical encyclopaedia in eight books, nearly seven of which are extant. See [Jones 1986], [Cuomo forthcoming].

most favorable case — is often considered the most competent mathematician in Late Antiquity, and Jones is his most careful contemporary reader. It is thus worth noting how Jones introduces his subject:

"*In the later Hellenistic period, after several hundred years of progress, the main stream of Greek mathematics, synthetic geometry, experienced a deep and permanent decline. The subject did not stop being studied and taught, but original discoveries became less and less frequent and important . . .*

Pappus of Alexandria is the first author in this degenerate tradition of whom we have substantial writings on higher geometry" [Jones 1986, p. 1].

Wary of teleological readings of the past, many historians would probably react instinctively against such terms as "decline" or "degeneration". Yet Jones' judgment is inescapable. Something did happen at the end of the Hellenistic period, and "decline" is the term which comes to mind. My purpose in this article, therefore, is not to try to show how original Late Antiquity was — for it was not. It was deeply conservative. Yet, I shall argue, it still had a real contribution to make, if inadvertently: it developed a new project which differed qualitatively from that of early mathematics and which shaped the future history of mathematics. The paradox is that such a change came about without any intention, on the part of Late Ancient mathematicians, to change their mathematics, and my argument is that in certain circumstances, and especially inside mathematics, conservatism can act as a force for change (we shall need, however, to specify precisely the intended sense of this "conservatism" later on in the article).

So, to start, one should notice that Late Ancient texts often take the form of commentaries, and even when they are not commentaries they often are what I call "deuteronomic texts". Late Antiquity is the age of new editions, epitomes, and encyclopedic collections; later, during the Middle Ages — which, in this respect essentially pursue trends already discernible in Late Antiquity — another kind of deuteronomic text was added, namely translations — into Syriac, Arabic and Latin.[3] The entire period from Late Antiquity to the Middle Ages is the age of scholia and marginalia. All such

[3] One can add at least one case of translation inside Greek culture itself, namely the translation of some works by Archimedes (*Sphere and Cylinder*, *Method*), from the original Doric dialect, into the dominant *koine* dialect.

texts are "deuteronomic": they explicitly start from an established text (or texts, in the plural), and aim at producing a new text, which reenacts the earlier text (as in a translation or a new edition), or uses it in more radical ways (an epitome or an encyclopedia). At least from the modern point of view, commentary is the most important kind of deuteronomic text, because it is also the most ambitious: it is the deuteronomic text standing on its own, apart from the original text; but in this article I shall stress commonalties among different kinds of deuteronomic texts rather than distinctions. In general, I shall see the commentary form as key to the understanding of deuteronomic texts.

Generally speaking, commentators are not highly regarded by modern authors, and they are often referred to by such pejorative terms as "pedantic" or "scholastic".[4] And once again, I do not wish to contest this characterization: my purpose is to identify in detail what makes an author appear "scholastic". I shall then argue that such "scholasticism" may have real historical and philosophical significance and is, in fact, the vehicle through which conservatism can act as a force for change.

2. What is "Scholasticism"?

What do we mean by "scholastic" or "pedantic"? I shall now try to unpack such concepts with the aid of examples. I shall argue that there are several things we may mean by such terms, all closely linked. After we have seen some of these possible meanings, we shall try to investigate the possible link: what exactly do commentators tend to do, which earns them their pejorative epithets?

(a) Scholia and "Vertical Pedantry"

First, one thing commentators do is *explain the obvious.* This is vertical pedantry: they dig too deep. Of course, the "obvious" is difficult to define, and it is clear that shaped by its distinct mathematical education each mathematical culture will consider different things as "obvious",[5] but it *is* necessary to stop somewhere in a proof, otherwise Carroll's well-known paradox ensues [Carroll 1895]: in this paradox, to prove that Q derives from P you must prove that P yields Q, and then you need to prove that

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[4] See *e.g.* [Knorr 1989, p. 238–239, 812–816].

[5] See especially [Goldstein 1995] on the historical variability of such seemingly neutral concepts as "the obvious".

from P, and P yields Q, Q derives, and so on *ad infinitum*. This is not an empty philosophical worry: this type of regress may be called “the scholiasts’ regress”, and it is well-attested historically. Take, for instance, the final proposition of the first book of Archimedes’ *Sphere and Cylinder*. The following quotation has been taken out of the text itself (*i.e.* not from a separate commentary), but it is clear that Archimedes is not the sole responsible for this text. Scholia have accumulated and entered the Archimedean text, so that the deuteronomic text directly manipulates the original — a crucial point to which we shall have to return. This is how it works explicitly[6] (Fig. 1).

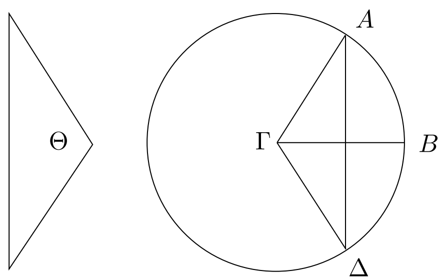

Figure 1

[A conclusion of Archimedes’ line of reasoning:]

“*Therefore the figure inscribed in the sector, too, is greater than the cone* Θ*; which is impossible.*”

From the mathematical context, impossibility can be seen directly: in fact, it derives immediately from the proposition just preceding this one. To note this fact, some scholiast added the following comment:

“*for it has been proved in the* [*proposition*] *above that it is smaller than a cone of this kind.*”

This is the first pedantic note, the first explanation of the obvious. Now the same scholiast, a minute later, or another one, a century later, hastens to add:

[6] [Heiberg 1910, 162.25–164.11]. It should be clear that much modern editorial work – and much subjective judgement – is implicit in any reference to “Archimedes” or to “the scholiast”. Still, this is a case where the two terms seem warranted, on linguistic and other grounds.

"*that is* [*a cone*] *having* [*as*] *a base a circle whose radius is equal to the line drawn from the vertex of the segment to the circumference of the circle (which is* [*the*] *base of the segment), and* [*as*] *a height the radius of the sphere.*"

This is of course the standard description — scholiasts care about standard descriptions, a fact to which we shall return. But wait – is this really the cone we have here? Yes, assures the scholiast:

"*and this is the said cone* Θ."

Or is it? Wonders the same scholiast, or yet another one, yet another century later: We must say why!

"*for it has both*: [*as*] *a base, a circle equal to the surface of the segment, that is* [*equal*] *to the said circle, and a height equal to the radius of the sphere.*"

This is, then, what one calls "pedantic": explaining explicitly what should be understood implicitly — a process to which there is in principle no end and which therefore can make the pedant look not only dim-witted, but also absurd. At any rate, this is clearly one type of pedantry, vertical pedantry: digging too deep. Apparent absurdities of the type quoted above are relatively infrequent (though more can be easily added: *e.g.*, in the same book, Proposition 13, [Heiberg 1910, 56.10–24], because scholia become recursive only when an original scholion becomes part of the transmitted text — a common but far from universal phenomenon; while marginalia to marginalia are less common. However, this is almost the most common type of scholion we find in mathematical works: a brief, essentially trivial, mathematical explication, showing why a derivation works — a question which in principle could always be raised and therefore was less frequently raised by the original Greek mathematicians.[7]

(b) "Horizontal Pedantry"

So far I have described what I call "vertical pedantry", where you dig too deep. Another, related type of pedantry is "horizontal pedantry": digging too wide. Just as one can go on proving obvious things, anterior to the proof, so one can go on proving implied things, posterior to the

[7] See [Knorr 1996, p. 222–242], for a full discussion of this phenomenon of such, usually very brief, explications to arguments (*e.g.* in the form of cross-reference – on which more below).