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A SPECTRAL RESOLUTION OF THE LARGE SIEVE

BY OLIVIER RAMARÉ

ABSTRACT. — Let $\varphi = (\varphi_n)_{n \leq N}$ denote a sequence of complex numbers. The main object of this paper is the quadratic form

$$V(\varphi, Q) = \sum_{Q < q \leq 2Q} \sum_{a \bmod^* q} \left| \sum_{n=1}^N \varphi_n e^{2i\pi na/q} \right|^2,$$

where $a \bmod^* q$ denotes an integer variable a that goes through the points of $\{1, \dots, q\}$ that are prime to q . The real parameters N and Q are assumed to be large. The eigenvalues of $V(\cdot, Q)$ are well understood when $Q = o(\sqrt{N})$ and $V(\varphi, Q)$ behaves like a Riemann sum when $N = o(Q)$. The behavior in the range $Q \in [\sqrt{N}, 100N]$ is the object of our queries. In particular, we present a full spectral analysis when $Q \geq N^{3/4}$ in terms of the eigenvalues of a one-parameter family of nuclear difference operators. Among the consequences, we show that $V(\varphi, Q)/(Q^2 \sum_n |\varphi_n|^2)$ stays between two positive constants uniformly in φ , when N/Q stays similarly between two positive constants, and that (a smoothed version of) the quadratic form $V(\cdot, Q)$ may stay *away* from the diagonal form $\varphi \mapsto (6/\pi^2)Q^2 \sum_n |\varphi_n|^2$ when N/Q stays between two positive constants, although only on a vector space of φ of positive but small dimension.

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