ISSN 0012-9593

quatrième série - tome 58 *fascicule 6* *novembre-décembre 2025*

ANNALES SCIENTIFIQUES de L'ÉCOLE NORMALE SUPÉRIEURE

Ted CHINBURG, Robert GURALNICK & David HARBATER

Erratum to "The local lifting problem for actions of finite groups on curves"

SOCIÉTÉ MATHÉMATIQUE DE FRANCE

Annales Scientifiques de l'École Normale Supérieure

Publiées avec le concours du Centre National de la Recherche Scientifique

Responsable du comité de rédaction / *Editor-in-chief*

Javier Fresán

Publication fondée en 1864 par Louis Pasteur

Continuée de 1872 à 1882 par H. Sainte-Claire Deville
de 1883 à 1888 par H. Debray
de 1889 à 1900 par C. Hermite
de 1901 à 1917 par G. Darboux
de 1918 à 1941 par É. Picard
de 1942 à 1967 par P. Montel

Comité de rédaction au 19 mai 2025

S. Cantat
G. Carron
Y. Cornulier
B. Fayad
D. Häfner
D. Harari
Y. Harpaz
C. Imbert
A. Keating
G. Miermont
S. Riche
P. Shan

Rédaction / *Editor*

Annales Scientifiques de l'École Normale Supérieure,
45, rue d'Ulm, 75230 Paris Cedex 05, France.
Tél. : (33) 1 44 32 20 88.
Email : annales@ens.fr

Édition et abonnements / *Publication and subscriptions*

Société Mathématique de France
Case 916 - Luminy
13288 Marseille Cedex 09
Tél. : (33) 04 91 26 74 64
Email : abonnements@smf.emath.fr

Tarifs

Abonnement électronique : 494 euros.
Abonnement avec supplément papier :
Europe : 694 €. Hors Europe : 781 € ($ 985). Vente au numéro : 77 €.

© 2025 Société Mathématique de France, Paris

En application de la loi du 1er juillet 1992, il est interdit de reproduire, même partiellement, la présente publication sans l'autorisation de l'éditeur ou du Centre français d'exploitation du droit de copie (20, rue des Grands-Augustins, 75006 Paris).
All rights reserved. No part of this publication may be translated, reproduced, stored in a retrieval system or transmitted in any form or by any other means, electronic, mechanical, photocopying, recording or otherwise, without prior permission of the publisher.

ISSN 0012-9593 (print) 1873-2151 (electronic)

Directrice de la publication : Isabelle Gallagher
Périodicité : 6 nos / an

Ann. Scient. Éc. Norm. Sup.
4ᵉ série, t. 58, 2025, p. 1581 à 1583

ERRATUM TO "THE LOCAL LIFTING PROBLEM FOR ACTIONS OF FINITE GROUPS ON CURVES"

BY TED CHINBURG, ROBERT GURALNICK AND DAVID HARBATER

ABSTRACT. – We correct Theorem 1.5 in our paper "The local lifting problem for actions of finite groups on curves," *Ann. Sci. Éc. Norm. Supér.* (4) **44** (2011), 537–605.

RÉSUMÉ. – Nous corrigeons le théorème 1.5 de notre article "The local lifting problem for actions of finite groups on curves," *Ann. Sci. Éc. Norm. Supér.* (4) **44** (2011), 537–605.

With the notation of [1], the goal of this note is to explain how examples found by B. Weaver in [2] lead to the following corrected form of Theorem 1.5 of [1].

THEOREM 1.5. – *For k algebraically closed of characteristic p, a group G is an almost KGB group for k if and only if it is an almost Bertin group k. Such G are exactly those isomorphic to a group of one of the following kinds:*

i. *A cyclic group.*

ii. *A dihedral group D_{2p^n} of order $2p^n$ for some $n \geq 1$.*

iii. *The alternating group A_4 when $p = 2$.*

iv. *A generalized quaternion group Q_{2^m} of order 2^m for some $m \geq 4$ when $p = 2$.*

The original statement of Theorem 1.5 of [1] had also included the groups $\mathrm{SL}_2(\mathbb{Z}/3)$ and Q_8 when $p = 2$. We had shown that every almost KGB group is on the resulting larger list, and we claimed that the converse holds as well. But examples constructed by B. Weaver in Chapter 7 of [2] showed that $\mathrm{SL}_2(\mathbb{Z}/3)$ and Q_8 are not in fact almost KGB groups when $p = 2$, and we thank him for pointing this out.

Theorem 1.5 above and Theorem 1.2 of [1] yield the following (see [2, Prop. 7.2.11]).

The first author is the corresponding author. He was supported in part by NSF FRG Grants No. DMS-1360767 and No. DMS-2411702, and NSF SaTC grants No. CNS-1513671 and No. CNS-1701785. The second author was partially supported by NSF Grant DMS-1901595 and Simons Fellowship 609771. The third author was supported in part by NSF grants DMS-1805439 and DMS-2102987.

0012-9593/06/© 2025 Société Mathématique de France. Tous droits réservés
doi:10.24033/asens.2634

COROLLARY. – *For $k = \overline{k}$ of characteristic p, the notions of being a KGB group, a Bertin group, an almost KGB group and an almost Bertin group coincide.*

We now list the results stated in [1] that are not correct and indicate how to correct them.

1. Lemma 15.15 and Corollary 15.16 are not correct. In §15, the field k is assumed to be quasi-finite of characteristic p and is not assumed to be algebraically closed (see Hypothesis 15.1). The error in Lemma 15.15 arises from the statement just after equation (15.33) that if $\lambda \in k$ then there is a $\zeta \in K$ so $\zeta^2 - \zeta^4 = \lambda$. This is not true, for example, if k has order 2. As a result, Corollary 15.16 does not hold. The error in Lemma 15.15 concerns only the claim that $i_0 = 1$ under the hypotheses of Proposition 15.6(i). The existence of examples in which i_0 is arbitrarily large accounts for why Q_8 and $\mathrm{SL}_2(3)$ are not almost Bertin groups.

2. The proof of Proposition 15.6(i) relied on Corollary 15.16 and is not correct. Proposition 15.6(i) should be changed to the following statement. The integer i_{n-1} is even unless G is the quaternion group of order 8. If i_{n-1} is odd, then the Bertin obstruction of $\phi : G \to \mathrm{Aut}_k(k[[t]])$ does not vanish, and the lower ramification filtration of G is has the form

$$G_0 = \cdots = G_{i_0} \neq G_{i_0+1} = \cdots = G_{3i_0} \neq G_{3i_0+1} = \{e\},$$

where G_{i_0+1} is the center of G. The proof of this result given on page 589 is correct except for the fact that one cannot say that $i_0 = 1$ due to the error in Lemma 15.15.

3. The statement of part (ii) of Proposition 15.6 is correct, but the proof needs to be corrected because it also relied on Corollary 15.16. To fix it, we suppose as in part (ii) of Proposition 15.6 that i_{n-1} is even. One needs to show that not all of hypotheses (i), (ii) and (iii) of Lemma 15.13 can hold. If all these hypotheses hold, then $n = 2$ and $c(1) = 2i_0$, but we cannot say $i_0 = 1$ due to the error in Corollary 15.16. However, Corollary 15.5 shows that i_{n-1} is even if and only if $c(n-1)$ is odd. Since $n = 2$ and $c(n-1) = c(1) = 2i_0$ is even, this is impossible since i_{n-1} even. The rest of the proof of part (ii) of Proposition 15.6 now proceeds as in [1].

4. Corollary 15.7 is not correct. It does not follow from work of Serre and Fontaine, and it is not implied by the corrected version of Proposition 15.6. We never use Corollary 15.7 elsewhere in [1].

5. The assertion of Proposition 17.2(i) is correct but the proof should be adjusted in the following way. Instead of referring to Proposition 15.6(iii), one should use the corrected form of Proposition 15.6(i,ii) indicated in items #2 and #3 above. Proposition 17.2(ii) and Corollary 17.3 are not correct because of the examples found by B. Weaver [2] showing that Q_8 and $\mathrm{SL}_2(3)$ are not almost KGB groups for k when $p = 2$. The error in the arguments of [1] concerning Proposition 17.2(ii) and Corollary 17.3 is that Proposition 15.6(i) is not correct in its original form, for the reasons described above. The incorrect proof of Proposition 17.2(ii) relied on the incorrect Proposition 15.6(i) (rather than on Proposition 15.6(iii)), and the proof of Corollary 17.3 is incorrect because if relied on Proposition 17.2(ii). Proposition 17.2(iii) and its proof are correct. Proposition 16.1 holds since the proofs carry through with the correction in item 2 above to part (i) of Proposition 15.6. Note that Lemma 17.5

relies only on part (ii) of Proposition 15.6 rather than on part (i) of Proposition 15.6, and part (ii) of Proposition 15.6 is correct.

6. Theorem 1.5 should exclude $\mathrm{SL}_2(3)$ and Q_8 from the list of almost Bertin groups when $p = 2$.

BIBLIOGRAPHY

[1] T. Chinburg, R. Guralnick, D. Harbater, The local lifting problem for actions of finite groups on curves, *Ann. Sci. Éc. Norm. Supér.* **44** (2011), 537–605.

[2] B. Weaver, Non-abelian groups of order eight and the local lifting problem, Ph.D. Thesis, University of Virginia, 2018, doi:10.18130/V3MK6579K.

(Manuscrit reçu le 31 mars 2025 ;
accepté le 12 juin 2025.)

Ted Chinburg
Department of Mathematics
University of Pennsylvania
Philadelphia, PA 19104-6395, USA
E-mail: ted@math.upenn.edu

Robert Guralnick
Department of Mathematics
University of Southern California
Los Angeles, CA 90089-2532, USA
E-mail: guralnic@usc.edu

David Harbater
Department of Mathematics
University of Pennsylvania
Philadelphia PA 19104-6395, USA
E-mail: harbater@math.upenn.edu