

# CENTRAL SHEAVES ON AFFINE FLAG VARIETIES

Pramod N. Achar & Simon Riche



Panoramas et Synthèses

Numéro 64

2025

SOCIÉTÉ MATHÉMATIQUE DE FRANCE

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*Vente au numéro* : 79 € (\$ 119)

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ISSN 1272-3835

ISBN 978-2-37905-216-3

Directrice de la publication : Isabelle Gallagher

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P.A. was supported by NSF Grant Nos. DMS-1802241 and DMS-2202012. This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (S.R., grant agreements No. 677147 & 101002592).

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*Classification mathématique par sujets.* (2020) — 20G05, 14F05, 14M15.

*Keywords and phrases.* — Perverse sheaves, reductive algebraic groups, affine Grassmannians, affine flag varieties, nearby cycles, loop groups, Springer resolution.

*Mots-clés et phrases.* — Faisceaux pervers, groupes algébriques réductifs, Grassmanniennes affines, variétés de drapeaux affines, cycles proches, groupes de lacets, résolution de Springer.

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# CENTRAL SHEAVES ON AFFINE FLAG VARIETIES

Pramod N. Achar & Simon Riche

**Abstract.** — In this book we explain the construction (due to Gaitsgory) of the “central sheaves” on the affine flag variety of a reductive algebraic group associated with the representations of the Langlands dual group. These objects are the categorical counterpart of the Bernstein description of the center of the affine Hecke algebra. They play a crucial role in many constructions related to the geometric Langlands program, some of which have important applications to the representation theory of reductive algebraic groups and associated quantum groups. We also explain the proof of the main properties of these objects (in particular the filtration by Wakimoto sheaves), for arbitrary coefficients. Finally, we present in detail the construction of an equivalence of categories due to Arkhipov-Bezrukavnikov relating certain derived categories of constructible sheaves on the affine flag variety and coherent sheaves on the Springer resolution of the dual group, in which central sheaves play a key role.

**Résumé.** (Faisceaux centraux sur les variétés de drapeaux affines) — Dans ce livre nous expliquons la construction (due à Gaitsgory) des « faisceaux centraux » sur la variété de drapeaux affine d’un groupe algébrique réductif associés aux représentations du groupe dual au sens de Langlands. Ces objets sont le pendant « catégorique » de la description de Bernstein du centre de l’algèbre de Hecke affine. Ils jouent un rôle crucial dans de nombreuses constructions liées au programme de Langlands géométrique, dont certaines ont des applications importantes en théorie des représentations des groupes algébriques réductifs et des groupes quantiques associés. Nous expliquons également la preuve des propriétés principales de ces objets (notamment la filtration par les objets de Wakimoto), pour des coefficients arbitraires. Enfin, nous présentons en détail la construction d’une équivalence de catégories due à Arkhipov-Bezrukavnikov qui relie certaines catégories dérivées de faisceaux constructibles sur la variété de drapeaux affine et de faisceaux cohérents sur la résolution de Springer du groupe dual, et dans laquelle les faisceaux centraux jouent un rôle crucial.



# CONTENTS

|                                                                |    |
|----------------------------------------------------------------|----|
| <b>Introduction</b> .....                                      | 1  |
| 0.1. Overview .....                                            | 1  |
| 0.1.1. Langlands duality philosophy .....                      | 1  |
| 0.1.2. Geometric Satake equivalence .....                      | 1  |
| 0.1.3. History of the geometric Satake equivalence .....       | 2  |
| 0.1.4. Gaitsgory's central functor .....                       | 5  |
| 0.1.5. Arkhipov-Bezrukavnikov equivalence(s) .....             | 5  |
| 0.2. Contents .....                                            | 6  |
| 0.2.1. Part I .....                                            | 6  |
| 0.2.2. Part II .....                                           | 8  |
| 0.2.3. Part III .....                                          | 9  |
| 0.3. Some conventions .....                                    | 9  |
| 0.4. Acknowledgements .....                                    | 10 |
| <br><b>Part I. General theory of central sheaves</b> .....     | 11 |
| <br><b>1. Review of the geometric Satake equivalence</b> ..... | 13 |
| 1.1. Some technical preliminaries .....                        | 13 |
| 1.1.1. Ind-schemes .....                                       | 13 |
| 1.1.2. Attractors and repellers .....                          | 16 |
| 1.1.3. Attractors and repellers for ind-schemes .....          | 19 |
| 1.2. Affine Grassmannians .....                                | 20 |
| 1.2.1. Definition and representability .....                   | 21 |
| 1.2.2. $\mathbb{G}_m$ -actions via cocharacters .....          | 25 |
| 1.2.3. Semi-infinite orbits .....                              | 29 |
| 1.3. Main sheaf-theoretic constructions .....                  | 31 |
| 1.3.1. Sheaves and convolution .....                           | 32 |
| 1.3.2. The fiber functor .....                                 | 34 |
| 1.3.3. Fusion product and the commutativity constraint .....   | 35 |
| 1.3.4. Monoidal structure on total cohomology .....            | 37 |
| 1.3.5. Statement .....                                         | 38 |
| 1.4. Overview of the construction of the group scheme .....    | 39 |
| 1.4.1. Tannakian formalism .....                               | 39 |
| 1.4.2. The case of characteristic-0 coefficients .....         | 40 |
| 1.4.3. Projective generators .....                             | 41 |
| 1.4.4. Construction of the group scheme .....                  | 42 |
| 1.4.5. Identification of the group scheme .....                | 43 |

|                                                                                |           |
|--------------------------------------------------------------------------------|-----------|
| 1.5. Some complements .....                                                    | 45        |
| 1.5.1. Standard and costandard spherical perverse sheaves .....                | 45        |
| 1.5.2. More on costandard spherical perverse sheaves .....                     | 46        |
| <b>2. Central sheaves .....</b>                                                | <b>51</b> |
| 2.1. Preliminaries on torsors and quotients .....                              | 51        |
| 2.1.1. Schemes as sheaves .....                                                | 52        |
| 2.1.2. Torsors and principal bundles .....                                     | 53        |
| 2.1.3. Trivializations over certain base schemes .....                         | 55        |
| 2.1.4. Descent and associated bundles .....                                    | 58        |
| 2.2. Global affine Grassmannians .....                                         | 61        |
| 2.2.1. The affine Grassmannian and the affine flag variety .....               | 61        |
| 2.2.2. Moduli interpretation .....                                             | 62        |
| 2.2.3. Global group schemes and global affine Grassmannians .....              | 66        |
| 2.2.4. Example: the first fundamental coweight for $GL(n)$ .....               | 71        |
| 2.3. Iterated global affine Grassmannians .....                                | 73        |
| 2.3.1. Overview .....                                                          | 73        |
| 2.3.2. Beilinson–Drinfeld group schemes .....                                  | 74        |
| 2.3.3. Beilinson–Drinfeld affine Grassmannian .....                            | 75        |
| 2.3.4. Iterated global affine Grassmannians .....                              | 76        |
| 2.3.5. Principal bundles and representability .....                            | 78        |
| 2.3.6. More principal bundles over $\mathbf{Gr}_G(S_1, \dots, S_n)$ .....      | 82        |
| 2.3.7. Special and generic fibers .....                                        | 82        |
| 2.4. Nearby cycles on iterated global affine Grassmannians .....               | 85        |
| 2.4.1. Equivariant derived categories .....                                    | 85        |
| 2.4.2. $\mathbb{G}_m$ -actions .....                                           | 86        |
| 2.4.3. Twisted products and convolution .....                                  | 87        |
| 2.4.4. Nearby cycles .....                                                     | 89        |
| 2.4.5. The functor $Z$ .....                                                   | 90        |
| 2.5. Global affine Grassmannians and nearby cycles for $G_C$ .....             | 92        |
| 2.5.1. Global group scheme and global affine Grassmannian .....                | 92        |
| 2.5.2. Beilinson–Drinfeld group schemes and affine Grassmannian .....          | 93        |
| 2.5.3. Iterated global affine Grassmannians .....                              | 93        |
| 2.5.4. Nearby cycles .....                                                     | 94        |
| <b>3. Braiding compatibilities .....</b>                                       | <b>95</b> |
| 3.1. Overview of central functors .....                                        | 95        |
| 3.2. Centrality isomorphism .....                                              | 97        |
| 3.2.1. Convolution with central sheaves .....                                  | 97        |
| 3.2.2. Some consequences .....                                                 | 100       |
| 3.3. Variants for $G_C$ .....                                                  | 103       |
| 3.3.1. Convolution as nearby cycles .....                                      | 103       |
| 3.3.2. Comparison with the fusion product .....                                | 105       |
| 3.3.3. Monoidal structure on total cohomology via nearby cycles .....          | 106       |
| 3.3.4. Monoidal structure on total cohomology via equivariant cohomology ..... | 107       |
| 3.4. The central functor is monoidal .....                                     | 109       |
| 3.4.1. Monoidality isomorphism .....                                           | 109       |

|                                                                                                |            |
|------------------------------------------------------------------------------------------------|------------|
| 3.4.2. Projection to $\mathrm{Gr}_G$ .....                                                     | 111        |
| 3.5. Compatibility of commutativity .....                                                      | 113        |
| 3.5.1. Statement .....                                                                         | 113        |
| 3.5.2. Affine Grassmannians over a 2-dimensional base .....                                    | 113        |
| 3.5.3. Convolution .....                                                                       | 115        |
| 3.5.4. Relation with the fusion product .....                                                  | 115        |
| 3.5.5. Restriction to subvarieties of $C^2$ .....                                              | 116        |
| 3.5.6. Iterated cleanness .....                                                                | 116        |
| 3.5.7. Study of nearby cycles over $C^2$ .....                                                 | 117        |
| 3.5.8. Proof of Theorem 3.5.1 .....                                                            | 118        |
| 3.5.9. Alternative description of the commutativity constraint of the Satake<br>category ..... | 120        |
| <b>4. Central sheaves, Wakimoto sheaves, and the Satake equivalence .....</b>                  | <b>123</b> |
| 4.1. Standard sheaves, costandard sheaves, and their convolutions .....                        | 123        |
| 4.1.1. Affine Weyl group combinatorics .....                                                   | 123        |
| 4.1.2. Standard and costandard objects on $\mathrm{Fl}_G$ .....                                | 126        |
| 4.1.3. Convolutions of standard and costandard objects .....                                   | 129        |
| 4.1.4. Support of convolutions of standard and costandard objects .....                        | 130        |
| 4.2. Wakimoto sheaves .....                                                                    | 132        |
| 4.2.1. Wakimoto functors .....                                                                 | 132        |
| 4.2.2. Full faithfulness .....                                                                 | 133        |
| 4.2.3. Wakimoto functors and convolution .....                                                 | 134        |
| 4.3. Wakimoto filtrations .....                                                                | 134        |
| 4.3.1. Wakimoto functors and morphisms .....                                                   | 134        |
| 4.3.2. Wakimoto filtrations .....                                                              | 135        |
| 4.3.3. The associated graded functor .....                                                     | 136        |
| 4.3.4. Stability under direct summands .....                                                   | 138        |
| 4.4. Central sheaves admit Wakimoto filtrations .....                                          | 139        |
| 4.4.1. An existence criterion .....                                                            | 139        |
| 4.4.2. Central sheaves admit filtrations by Wakimoto sheaves .....                             | 141        |
| 4.5. Cohomology of Wakimoto-filtered perverse sheaves .....                                    | 142        |
| 4.5.1. Cohomology .....                                                                        | 142        |
| 4.5.2. Equivariant cohomology .....                                                            | 144        |
| 4.5.3. Cohomology with support .....                                                           | 146        |
| 4.6. Consequences .....                                                                        | 148        |
| 4.6.1. First consequences .....                                                                | 149        |
| 4.6.2. Monodromy and Wakimoto filtrations .....                                                | 153        |
| 4.6.3. Highest weight arrows .....                                                             | 154        |
| 4.6.4. Highest weight arrows, convolution, and commutativity .....                             | 156        |
| 4.7. Associated graded of the Wakimoto filtration and convolution .....                        | 159        |
| 4.7.1. Associated graded of a convolution .....                                                | 159        |
| 4.7.2. Monoidal structure on $\mathrm{Grad}_{X^\vee}^A$ .....                                  | 162        |
| 4.8. Comparison of the functors $\mathrm{Grad}$ and $\mathbf{F}$ .....                         | 164        |
| 4.8.1. Grading .....                                                                           | 164        |
| 4.8.2. Monoidal structures .....                                                               | 165        |

|                                                                                                         |         |
|---------------------------------------------------------------------------------------------------------|---------|
| <b>5. Combinatorial aspects and variant for étale sheaves</b>                                           | 169     |
| 5.1. Bernstein's description of the center of the affine Hecke algebra                                  | 169     |
| 5.1.1. Bernstein elements in the affine Hecke algebra                                                   | 169     |
| 5.1.2. The center of the affine Hecke algebra                                                           | 172     |
| 5.1.3. Relation with Kazhdan-Lusztig combinatorics                                                      | 172     |
| 5.2. Combinatorics of central sheaves                                                                   | 173     |
| 5.2.1. Description of the Grothendieck group of $D_I^b(\mathrm{Fl}_G, \mathbb{k})$                      | 173     |
| 5.2.2. Classes of central sheaves                                                                       | 175     |
| 5.3. Combinatorics of mixed central sheaves                                                             | 176     |
| 5.3.1. Étale central sheaves                                                                            | 176     |
| 5.3.2. Mixed complexes                                                                                  | 177     |
| 5.3.3. Associated graded of the weight filtration                                                       | 178     |
| 5.3.4. Mixed central sheaves                                                                            | 179     |
| <br><b>Part II. Application to the Arkhipov-Bezrukavnikov equivalence</b>                               | <br>181 |
| <b>6. The characteristic-0 Arkhipov-Bezrukavnikov equivalence</b>                                       | 183     |
| 6.1. Overview of the chapter                                                                            | 183     |
| 6.1.1. Categorifying the affine Weyl group and its antispherical module                                 | 183     |
| 6.1.2. Outline of the chapter                                                                           | 185     |
| 6.1.3. Conventions                                                                                      | 186     |
| 6.2. Coherent sheaves on the Springer resolution                                                        | 187     |
| 6.2.1. The basic affine space and its affine completion                                                 | 187     |
| 6.2.2. The Springer resolution and some variants                                                        | 190     |
| 6.2.3. Koszul complexes                                                                                 | 193     |
| 6.2.4. Equivariant coherent sheaves on $\tilde{\mathcal{N}}$                                            | 194     |
| 6.2.5. Equivariant coherent sheaves on $\tilde{\mathcal{N}}$ as a quotient                              | 195     |
| 6.3. Construction of the functor                                                                        | 197     |
| 6.3.1. Equivariant perverse sheaves on $\mathrm{Fl}_G$                                                  | 198     |
| 6.3.2. "Extension" of a restriction functor                                                             | 198     |
| 6.3.3. Starting point: central and Wakimoto sheaves                                                     | 200     |
| 6.3.4. Extending the functor to free coherent sheaves on $\hat{\mathcal{N}}_{\mathcal{X}}, \mathrm{I}$  | 201     |
| 6.3.5. Extending the functor to free coherent sheaves on $\hat{\mathcal{N}}_{\mathcal{X}}, \mathrm{II}$ | 206     |
| 6.3.6. Factorization through coherent sheaves on $\tilde{\mathcal{N}}$                                  | 208     |
| 6.4. Antispherical and Iwahori-Whittaker categories                                                     | 210     |
| 6.4.1. The antispherical category                                                                       | 210     |
| 6.4.2. The Iwahori-Whittaker category                                                                   | 210     |
| 6.4.3. Statement                                                                                        | 211     |
| 6.4.4. Some preliminaries                                                                               | 212     |
| 6.4.5. Proof of Theorem 6.4.2                                                                           | 214     |
| 6.5. Central sheaves and tilting Iwahori-Whittaker perverse sheaves                                     | 216     |
| 6.5.1. Statement                                                                                        | 216     |
| 6.5.2. Computing multiplicities                                                                         | 217     |
| 6.5.3. Propagation through tensor products                                                              | 218     |
| 6.5.4. Minuscule and quasi-minuscule coweights                                                          | 219     |
| 6.5.5. Extremal coweights                                                                               | 220     |
| 6.5.6. The regular quotient                                                                             | 221     |

|                                                                                                                                                                          |            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|
| 6.5.7. Description of the regular quotient .....                                                                                                                         | 223        |
| 6.5.8. Regularity of $n_0$ .....                                                                                                                                         | 227        |
| 6.5.9. Consequence for the stalks and costalks of central sheaves .....                                                                                                  | 228        |
| 6.5.10. The case of quasi-minuscule coweights .....                                                                                                                      | 229        |
| 6.5.11. Restriction to the regular orbit .....                                                                                                                           | 229        |
| 6.6. Proof of the equivalence .....                                                                                                                                      | 231        |
| 6.6.1. Statement .....                                                                                                                                                   | 231        |
| 6.6.2. Preliminaries .....                                                                                                                                               | 232        |
| 6.6.3. Proof of Theorem 6.6.1 .....                                                                                                                                      | 234        |
| 6.6.4. Application: indecomposability of $\mathcal{X}^{TW}(V)$ when $V$ is simple .....                                                                                  | 235        |
| <b>7. Complements .....</b>                                                                                                                                              | <b>237</b> |
| 7.1. t-structures .....                                                                                                                                                  | 237        |
| 7.1.1. Exceptional collections and associated t-structures .....                                                                                                         | 237        |
| 7.1.2. The exotic t-structure .....                                                                                                                                      | 238        |
| 7.1.3. Exotic and perverse t-structures .....                                                                                                                            | 240        |
| 7.1.4. Some consequences .....                                                                                                                                           | 241        |
| 7.2. Description of the regular quotient .....                                                                                                                           | 241        |
| 7.2.1. Support of simple exotic sheaves .....                                                                                                                            | 242        |
| 7.2.2. Induced equivalence for the regular nilpotent orbit .....                                                                                                         | 243        |
| 7.2.3. Consequence for the equivalence $\Phi^0$ .....                                                                                                                    | 245        |
| 7.3. A perverse description of equivariant coherent sheaves on $\mathcal{N}$ .....                                                                                       | 247        |
| 7.3.1. Statement .....                                                                                                                                                   | 247        |
| 7.3.2. Pushforward to the nilpotent cone .....                                                                                                                           | 248        |
| 7.3.3. Quotient by some simple exotic sheaves .....                                                                                                                      | 250        |
| 7.3.4. Proof of Theorem 7.3.1 .....                                                                                                                                      | 253        |
| <b>8. A modular Arkhipov-Bezrukavnikov equivalence for <math>GL(n)</math> .....</b>                                                                                      | <b>255</b> |
| 8.1. Coherent sheaves on the Springer resolution .....                                                                                                                   | 255        |
| 8.1.1. The basic affine space and its affine completion .....                                                                                                            | 256        |
| 8.1.2. The Springer resolution and some variants .....                                                                                                                   | 257        |
| 8.1.3. Koszul complexes .....                                                                                                                                            | 258        |
| 8.1.4. Equivariant coherent sheaves on $\tilde{\mathcal{U}}_{\mathbb{k}}$ .....                                                                                          | 260        |
| 8.1.5. Describing the category of equivariant coherent sheaves on $\tilde{\mathcal{U}}_{\mathbb{k}}$ as a quotient .....                                                 | 260        |
| 8.2. Construction of the functor .....                                                                                                                                   | 263        |
| 8.2.1. Definition of a functor on $\text{Coh}_{\text{fr, tilt}}^{G_{\mathbb{k}}^{\vee} \times T_{\mathbb{k}}^{\vee}}(\hat{\mathcal{U}}_{\mathcal{X}, \mathbb{k}})$ ..... | 263        |
| 8.2.2. Factorization through coherent sheaves on $\tilde{\mathcal{U}}_{\mathbb{k}}$ .....                                                                                | 267        |
| 8.3. Antispherical and Iwahori-Whittaker categories .....                                                                                                                | 268        |
| 8.4. Central sheaves and tilting Iwahori-Whittaker perverse sheaves .....                                                                                                | 268        |
| 8.4.1. Statement .....                                                                                                                                                   | 268        |
| 8.4.2. Proof of Theorem 8.4.1 .....                                                                                                                                      | 269        |
| 8.4.3. The regular quotient .....                                                                                                                                        | 270        |
| 8.4.4. Monodromy and the regular quotient .....                                                                                                                          | 272        |
| 8.5. Proof of the equivalence .....                                                                                                                                      | 275        |
| 8.5.1. Statement .....                                                                                                                                                   | 275        |
| 8.5.2. Preliminaries .....                                                                                                                                               | 276        |
| 8.5.3. Proof of Theorem 8.5.1 .....                                                                                                                                      | 277        |

|                                                                               |     |
|-------------------------------------------------------------------------------|-----|
| <b>Part III. Appendices</b> .....                                             | 279 |
| <b>9. Review of the main properties of nearby cycles</b> .....                | 281 |
| 9.1. Definition and basic properties .....                                    | 281 |
| 9.1.1. Definition .....                                                       | 281 |
| 9.1.2. Some basic properties .....                                            | 282 |
| 9.1.3. Monodromy .....                                                        | 286 |
| 9.2. Beilinson's construction of unipotent nearby cycles .....                | 286 |
| 9.2.1. The unipotent nearby cycles functor .....                              | 287 |
| 9.2.2. Beilinson's construction .....                                         | 287 |
| 9.3. Monodromic complexes .....                                               | 292 |
| 9.3.1. Definitions .....                                                      | 292 |
| 9.3.2. Monodromy .....                                                        | 293 |
| 9.3.3. Monodromic perverse sheaves .....                                      | 294 |
| 9.3.4. Monodromic complexes and nearby cycles .....                           | 297 |
| 9.4. Nearby cycles over a two-dimensional base .....                          | 298 |
| 9.4.1. Definition .....                                                       | 298 |
| 9.4.2. Compatibility with proper pushforward and smooth pullback .....        | 299 |
| 9.4.3. Comparison with iterated nearby cycles .....                           | 300 |
| 9.4.4. Iterated-clean perverse sheaves .....                                  | 302 |
| 9.4.5. Comparison with diagonal nearby cycles .....                           | 304 |
| 9.4.6. Products .....                                                         | 308 |
| 9.5. Nearby cycles for étale sheaves .....                                    | 310 |
| 9.5.1. Definition .....                                                       | 310 |
| 9.5.2. Monodromy action .....                                                 | 311 |
| 9.5.3. Monodromic étale sheaves .....                                         | 312 |
| 9.5.4. The case $\mathbb{k} = \overline{\mathbb{Q}}_\ell$ .....               | 314 |
| <b>10. Equivariant derived categories in families</b> .....                   | 317 |
| 10.1. Acyclic bundles .....                                                   | 317 |
| 10.1.1. Acyclic morphisms .....                                               | 318 |
| 10.1.2. Stiefel and Grassmannian schemes .....                                | 318 |
| 10.1.3. Embedding a group scheme in a general linear group .....              | 320 |
| 10.1.4. Equivariant acyclic resolutions .....                                 | 323 |
| 10.2. Definition of the equivariant derived category .....                    | 325 |
| 10.2.1. Categories associated with resolutions .....                          | 326 |
| 10.2.2. Definition of the equivariant derived category .....                  | 328 |
| 10.2.3. Basic properties .....                                                | 328 |
| 10.2.4. Sheaf functors .....                                                  | 331 |
| 10.3. Equivariant nearby cycles and monodromy .....                           | 332 |
| 10.3.1. The case of algebraic groups acting trivially on $\mathbb{A}^1$ ..... | 332 |
| 10.3.2. The case of group schemes .....                                       | 332 |
| <b>Bibliography</b> .....                                                     | 335 |
| <b>Index of notation</b> .....                                                | 345 |