

ON THE SPANISH SYMBOL ‘U’ FOR THOUSANDS: SOME ANSWERS

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Abstract. — The difficulties that Castilian medieval accountants had in working with large numbers and in doing basic mathematical operations with Roman numerals, led them to develop a special form of this system: the *cuenta castellana* (Castilian numeral system). Essential elements of this system were the *calderón* and the *cuento*. The *calderón* was a special symbol used in Spain for centuries to represent the thousand while *cuento* was a word with the meaning of million, but mainly used to express a quantity of small coinage. In this paper the origin, evolution and death of both terms are shown.

Résumé (Sur le symbole espagnol ‘U’ pour des milliers. Quelques réponses)

Les difficultés rencontrées par les comptables médiévaux castillans pour travailler avec de grands nombres et pour effectuer les opérations mathématiques de base avec les chiffres romains les ont amenés à développer une forme spéciale de ce système : le *cuenta castellana* (compte castillan). Les éléments essentiels de ce système étaient le *calderón* (chaudron) et le *cuento*. Le *calderón* était un symbole spécial utilisé en Espagne pendant des siècles pour représenter le millier tandis que *cuento* était un mot signifiant « millions », mais principalement utilisé pour exprimer une quantité de petite monnaie. Dans cet article, l’origine, l’évolution et la disparition des deux termes sont exposées.

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1. INTRODUCTION

The aim of this paper is to throw some light on the questions posed in the works that Cajori wrote about the use of the *calderón*, a special symbol for thousands resembling a U, in Spanish documents from the fifteenth to the nineteenth century. To understand the contradictions that he found in the use of this symbol, it is necessary to go back to the origin of the symbol, to read the works of Pérez de Moya, one of the most important books of arithmetic written in Spanish, *Diálogos de aritmética práctica y especulativa* (*Dialogs of practical and theoretical arithmetics*), to see how it evolved and arrived at the *Gaceta de Madrid* to meet its death. In our journey we will also find some peculiar characteristics of the *cuenta castellana*, Castilian numeral system, an extension of the Roman numeral system, that was in use in Castille for almost five hundred years.

Cajori wrote about the use of *calderón* at least on four occasions: in 1908 [Cajori 1908], in 1912 [Cajori 1912], in 1922 [Cajori 1922] and in 1928 [Cajori 1928, p. 60].

The first one [Cajori 1908, p. 14] describes it as special symbol of European origin resembling the capital letters U, or O open at the top, to designate “thousands” used by Spanish Americans in the writing of large numbers. Basically, on the next occasions, he rephrased the same text with slight variations.

The second one [Cajori 1912] describes one singularity of the old Spanish numeral system: the use of the letter *U* in place of the Roman letter *M*. Also, he found some variations in its form and its substitution for a different one, even in reprints of sixteenth century original documents, as in Muñoz y Gómez [1888].

The third one [Cajori 1922] introduces the term *calderón* to denote this symbol and reflects again the use of other symbols to represent the same quantity but without any known connection among them. Another two questions arise in this paper: its origin and relationship with a similar Italian symbol.

The fourth one [Cajori 1928, p. 60] is found in the entries 92 and 93 of his book *A History Of Mathematical Notations Vol I*, which are an extended version of his earlier work on the subject.

In brief, the questions that Cajori arises about the *calderón* are:

- Its real form.
- Its origin and the origin of its different versions.

In addition to answering these questions, we will write about the use of the word *cuento*, meaning a million, its symbol and its relationship with the Portuguese word *conto*.

Finally, the title of this paper is a tribute to the works of Florian Cajori.

2. THE CASTILIAN NUMERAL SYSTEM

At the end of the fourteenth century in Europe two numeral systems were in competition for use in commercial arithmetics: the Roman and the Hindu-Arabic numeral systems. Although the positional decimal numeral system is more efficient than the Roman from an algorithmic point of view or to represent large numbers, its implantation was not easy. On the one hand, the Roman numerals were so familiar to the people, that sixteenth century Germans called them “deutsche Zahlen”¹ [Menninger 1969/2011, p. 286], and many Spanish authors like: Gutiérrez de Gualda [Gutiérrez 1564, p. 1], Tejeda [de Tejeda 1546, p. 3], Yciar [de Yciar 1549, p. 1], Pérez de Moya [Pérez de Moya 1562, p. 16], Caramuel [Caramuelis 1670, Arithmetica I, p. 6] or Aznar [Aznar de Polanco 1727, p. 5] among others, called “*cuenta castellana*” (literally, Castilian account) an extension of the Roman numeral system. It was believed that Roman numbers were more difficult to forge than Hindu-Arabic numbers, which led to the requirement to use Roman numerals in all the official accounts [Menninger 1969/2011, p. 426].

Medieval accountants faced the following requirements:

- (1) They had to use the Roman numeral system.
- (2) They had to prevent their accounts from being counterfeited or misinterpreted.
- (3) They had to make the basic arithmetic operations such as addition and subtraction as easy as possible.

However, Castilian accountants had an additional problem: they had to work with large numbers.

Roman numerals were based on the principle of addition. The seven letters are used to represent numbers: I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, and M = 1,000 [Guitel 1975, p.203]; placing them side by side implied, generally, addition of their values. However, the M became widely used as a symbol for 1,000 only in the Middle Ages [Menninger 1969/2011, p. 245]. Until then, it was usual to represent 1,000 with a

¹ German Numbers.

stroke over an I, like $\bar{I}$, or by enclosing an I between a C and a $\mathcal{O}$, like $C\mathcal{O}$. If they needed bigger numbers, more C and $\mathcal{O}$ were added. So, 10,000 was written as $CC\mathcal{O}\mathcal{O}$ and 100,000 written as $CCC\mathcal{O}\mathcal{O}\mathcal{O}$. In the same way 500 was represented by cutting 1,000 in half, and 5,000, or 50,000 as the halves of 10,000 and 100,000: $\mathcal{I}\mathcal{O}$, and $\mathcal{I}\mathcal{O}\mathcal{O}$ respectively. There are not many examples of how Romans represented bigger numbers. Probably the best known is the incomplete inscription of “*Elogium*” of Dulus, in which the sign for 100,000 occurs twenty-three times (available on line at <https://bit.ly/36eZh6j>). This is a proof of the limitations of the Roman system. Pliny asserts that at one time 100,000 was the “last” number known to the Romans, [Cagnat 1890, p. 32] and [Menninger 1969/2011, p. 44]. Therefore, when the million became more current, they were forced to create a new symbol: $\boxed{I}$, for 100,000. So, $\boxed{X}$ was the symbol for 1,000,000 and $\boxed{C}$ for 10,000,000 [Menninger 1969/2011, p. 245]. A good description of the origin of the Roman numerals can be found in [Keyser 1988].

In this way a number like 31,415,926 in Roman numerals should be written something like this:

$$\overline{\overline{XXXI}} \overline{CDXV} DCCCCXXVI$$

or

$$\boxed{CCCCIV} CC\mathcal{O}\mathcal{O} \mathcal{I}\mathcal{O} DCCCCXXVI.$$

Both solutions are hard to read, and even harder to add or subtract without the help of an abacus. This problem is particularly evident in the Castilian accounting books because the official unit of account, in which all quantities must be expressed, is the *maravedí*, a small fractional unit in the Castilian currency system. In 1512, its value was a thirty-fourth part of a real [de Ortega 1512, p. 7] but soon it depreciated and by the eighteenth century the Spanish silver real also known as the Spanish dollar, the famous “piece of eight,” was equal to 680 *maravedís* [Pérez de Moya 1798, Book VI, p. 256] and therefore one real was equal to 85 *maravedís*. Considering this situation, it was not unusual that accountants used the Roman numerals for the accounting entries and the Hindu-Arabic numeral system for the mathematical operations, as we can see in a footnote made in a distribution document found in the Municipal Archive of Linares [Linares 1564] (Figure 1).

The solution to all requisites, including the additional problem that the first point is incompatible with the fourth, was the Castilian numeral system. The Castilian numeral system solved these problems by the conversion of the Roman numeral system in a sort of a positional numeral system,

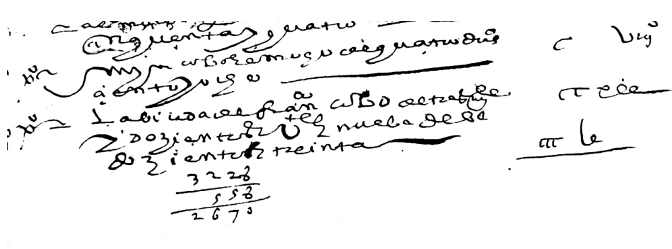


Figure 1. A footnote note showing a subtraction in a distribution document [Linares 1564] from 1564. The transcription states:²

	Fifty four	
Paid.	Martín Cobo, the lad, four ducats. One hundred and eight	CVIII
Paid.	The widow of Francisco Cobo	CCXXX
	three thousand two hundred twenty nine. She owes	
	two hundred and thirty	CCXXX
		CCLX
	3,228	
	558	
	2,670	

by splitting the large numbers to the powers of thousand with the inclusion of the symbols for the million, $\mathring{q}$, the letter q with a ring diacritic, and for the thousand, the *calderón*. In the Castilian numeral system our number should be written as:

XXXI $\mathring{q}$ CCCCXV U DCCCCXXVI.

which is clearly an easier way to handle numbers.

The first printed book that we have found containing the term and the description of the Castilian numeral system is [Gutiérrez 1564, p. 3]: “*Brief and useful art of the Castilian numerical system*” (“*Arte brebe y muy provechosa de cuenta castellana*”) written by Gutiérrez de Gualda and published in Toledo in 1539. In his book, Gutiérrez de Gualda describes its main characteristic, the use of special symbols:

- an m , to separate thousands;
- a $\mathring{q}$, to separate millions; and
- a dot ‘.’ to indicate empty spaces in the Roman numeral system.

² *Cinquenta y quatro*
Pagó. Martín Cobo, el mocho, de quatro ducados. Çiento y ocho CVIII
Pagó. La biuda de Françisco Cobo CCXXX
tres mill y dozientos e veinte e nueve. Debe
dozientos y treinta CCXXX
CCLX
3,228
558
2,670