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Microlocal Analysis in Hyperbolic Dynamics and Geometry

Thibault LEFEUVRE

with a contributed chapter by Yann Chaubet

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`www.ams.org`

Tarifs

Vente au numéro : 93 € ($ 140)

Des conditions spéciales sont accordées aux membres de la SMF.

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ISSN 1284-6090

ISBN 978-2-37905-221-7

Directrice de la publication : Isabelle GALLAGHER

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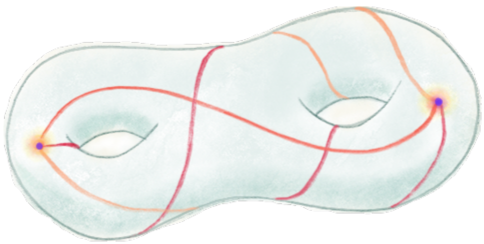

Société Mathématique de France 2025

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Mathematical Subject Classification (2010). — 35S05, 35P05, 35P99, 35R30, 37-02, 37C30, 37D20, 37D30, 53C24, 53D25, 53C29, 58J40.
Keywords. — Microlocal analysis, hyperbolic flows, spectral theory, inverse problems, geodesic flows, rigidity.
Mots-clefs. — Analyse microlocale, flots hyperboliques, théorie spectrale, problèmes inverses, flot géodésique, rigidité.

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Abstract. **—** The statistical properties of hyperbolic dynamical systems—such as ergodicity and mixing—can be studied through spectral theory, in particular via anisotropic Sobolev spaces of distributions. In settings where the geodesic flow exhibits hyperbolic features, rigidity phenomena in Riemannian geometry—showing that certain spectral or geometric invariants determine the underlying geometry—can likewise be addressed using microlocal analysis. This book offers a comprehensive introduction to microlocal analysis and its applications to hyperbolic dynamics and Riemannian rigidity. It is intended for graduate students and researchers seeking to familiarize themselves with these powerful techniques.

Résumé **(*Analyse micro-locale en dynamique hyperbolique et en géométrie*). —** Les propriétés statistiques des systèmes dynamiques hyperboliques, telles que l'ergodicité et le mélange, peuvent être étudiées à l'aide de la théorie spectrale, en particulier via les espaces de Sobolev anisotropes de distributions. Dans des contextes où le flot géodésique présente des caractéristiques hyperboliques, les phénomènes de rigidité en géométrie riemannienne, qui montrent que certains invariants spectraux ou géométriques déterminent la géométrie sous-jacente, peuvent également être abordés à l'aide de l'analyse microlocale. Cet ouvrage offre une introduction complète à l'analyse microlocale et à ses applications à la dynamique hyperbolique et à la rigidité riemannienne. Il s'adresse aux étudiants de troisième cycle et aux chercheurs qui souhaitent se familiariser avec ces techniques puissantes.

À la mémoire de mon père

“In the East they swore that by three sides
was the decent way across a square.”

T. E. Lawrence, *Seven Pillars of Wisdom*

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FOREWORD

I initiated this book project during the second Covid lockdown in France, in the fall of 2020, at a time when the world seemed to have come to a halt. Universities, libraries, and offices were closed, and academic life had largely shifted to virtual platforms. With only online seminars taking place and travel restrictions in effect, I found myself with an unexpected amount of uninterrupted time for reflection and writing. My initial goal was to produce a concise, 100-page survey on geometric inverse problems and microlocal analysis, intended as a reference for researchers and graduate students interested in these topics. However, I must confess that I largely failed in this task, as the so-called survey quickly grew out of control and exceeded 300 pages. I then decided to turn it into book which, if you are reading these lines, is now likely in your hands.

Beyond the unusual circumstances of its inception, this book also owes much to the series of lectures I delivered on microlocal analysis and hyperbolic dynamics at Sorbonne Université for master's students and at the Collège de France between 2021 and 2023. These lectures provided an opportunity to refine many of the ideas presented here and to shape their exposition in a way that would be accessible to a broad mathematical audience. Later, I had the privilege of further developing these topics in courses and seminars at Northwestern University, the University of Zurich, Fortaleza, and Montevideo. I am deeply grateful to the institutions and individuals who extended these invitations. Some of these lectures were recorded and are available online on my webpage. I will also maintain on the same page a list of errata.

I am grateful to all those who contributed, in one way or another to this book, whether in informal discussions or by providing me with proofs or exercises. I thank Louis Beaufort, Jan Bohr, Colin Guillarmou, Hedong Hou, Tristan Humbert, Maxime Ingremeau, Alessandro Morescalchi, Sebastián Muñoz-Thon, Gabriel Paternain, Amie Wilkinson and Yuhui Zhu for valuable feedback on this manuscript. This book project was also made possible through the support of the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant agreement no. 101162990—ADG).

I am deeply indebted to my long-time collaborator, Mihajlo Cekić, for the fruitful projects we have worked on together, some of which are presented in the latter part of this book. He also graciously provided proofs for certain lemmas, for which I am sincerely grateful. I am thankful to Yann Chaubet for contributing a chapter on dynamical zeta functions and for his valuable feedback on earlier drafts. I would also like to

acknowledge the anonymous referees, whose meticulous review of the manuscript was nothing short of remarkable. Their extensive 48-page report (!), filled with detailed suggestions and constructive criticism, has undoubtedly helped improve the presentation and clarity of this book. On a more personal note, I owe a profound debt of gratitude to Marie, whose unwavering support and encouragement have been invaluable throughout this long journey—none of this would have been possible without you.

Finally, I dedicate this book to the memory of my father, who passed away during the final stages of its writing in October 2024. Though he will never hold it in his hands, I have no doubt he would have been proud. I hope this book stands as a small tribute to his memory.

Paris, October 2020 – June 2025.