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A COMPRESSIBLE MULTIFLUID SYSTEM WITH NEW PHYSICAL RELAXATION TERMS

BY DIDIER BRESCH AND MATTHIEU HILLAIRET

ABSTRACT. – In this paper, we study the propagation of density-oscillations in solutions to density-dependent compressible Navier Stokes system. As a consequence to this analysis, we derive rigorously a generalization of the one-velocity Baer-Nunziato model for multifluid flows. The derived model includes a new relaxation term, in the PDE that governs the volume fraction of the component fluids, that encodes the change of viscosity and pressure between them.

RÉSUMÉ. – Dans cet article, nous étudions la propagation d'oscillations de densité dans les solutions des équations de Navier-Stokes compressibles fluides à viscosité variable. Nous appliquons cette analyse à la dérivation rigoureuse d'un système de type Baer-Nunziatio pour les écoulements multi-fluide. Le modèle obtenu inclut de nouveaux termes de relaxation dans les équations sur les fractions volumiques des composants du mélange. Ces termes résultent des différences entre les lois de viscosité et de pression dans les différents composants.

1. Introduction

This article is devoted to the rigorous derivation of a multifluid system with one velocity but considering the change of vicosity/pressure between the components. This model generalizes the Baer-Nunziato system with one velocity, already justified rigorously in [3]. Compared with this classical system, it introduces a mixture viscosity in the momentum equation and includes a new relaxation term in the PDEs satisfied by the volume fractions. These new terms highlight a non trivial interaction of the pressure and viscosity jumps in the behavior of the mixture.

If we look at physicists books such as those written by M. Ishii and T. Hibiki (see [9]) or by D. Drew and S.L. Passman (see [5]), we understand well that it is not so easy to choose the averaging process that has to be used to derive appropriate multifluid systems and that formal closure assumptions are often required in the approaches therein. Following [4, 8], we propose in [3] an alternative approach based on first principles exclusively. In this former reference, we consider a mixture in which bubbles of several different viscous compressible fluids coexist. Assuming a simplified behavior of interfaces (mainly a behavior that implies

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continuity of the mechanical quantities such as velocity and normal stresses through interfaces) and also that the different species share the same viscosity/isentropic pressure law, we introduce extended densities and velocities unifying then all the component equations into a single compressible Navier Stokes equation. The coexistence of several fluids in the mixture is then recovered by the fact that the extended density jumps between values inside the range of densities for the different component fluids. Hence, we derive our multifluid system by assuming that the initial density oscillates at small space scale and by computing the associate homogenized system. More precisely, we introduce Young measures to analyze the oscillations of the initial density and we assume that these Young measures concentrate in a finite number of Dirac masses (corresponding to the densities of the component fluids in the mixture). We derive the multifluid system by computing an equation for the concentration of the Dirac masses and their weights in the Young measures. One key difficulty in this method is to prove that finite convex combinations of Dirac masses are preserved through time-evolution by the equation satisfied by Young measures. An intermediate issue is to identify a functional framework in which the compressible Navier Stokes equations are well-posed (including uniqueness of solution) and that enables to consider discontinuous densities. The method we develop in [3, 4, 8] is closely-related to homogenization problem in compressible fluid mechanics. On this topic, the interested reader is also referred to [13, Section 7] where the kinetic equation formulation that we obtain in [3] is proposed in terms of the cumulative distribution function and without characterization of the Young measures which gives the multifluid systems.

In this paper, we extend the previous analysis to the compressible Navier-Stokes equations with density-dependent viscosity in the one-dimensional in space setting. We first study an appropriate notion of solution that encompasses discontinuous densities. We then tackle the homogenization problem for these solutions and the derivation of a multifluid system. From the modeling viewpoint, this density-dependent framework corresponds to a mixture with species having different viscosities. We believe this paper emphasizes the robustness of our method. In particular, we generalize here previous results initiated by A.A. Amosov and A.A. Zlotnik, see [1] and references cited therein, and also results by D. Serre in [14] related to the case with constant viscosity.

We give now a formal statement of our main result. Let assume that the three-dimensional container Ω contains the mixture of two viscous compressible phases described by triplets density/velocity/pressure (ρ_+, u_+, p_+) and (ρ_-, u_-, p_-) respectively. Introducing $(\mu_\pm, \lambda_\pm)$ and $p_\pm$ the respective viscosities and pressure laws of the phases, we obtain that, for $i = +, -$ the triplet is a solution to the compressible Navier Stokes equations

$$\partial_t \rho_i + \operatorname{div}(\rho_i u_i) = 0,$$
$$\partial_t(\rho_i u_i) + \operatorname{div}(\rho_i u_i \otimes u_i) = \operatorname{div}\Sigma_i,$$

on its domain $\mathscr{F}_i(t)$, with the equation of state:

$$\Sigma_i = \mu_i(\nabla u_i + \nabla^\top u_i) + (\lambda_i \operatorname{div} u_i - p_i)\mathbb{I}_3,$$
$$p_i = p_i(\rho_i).$$

Neglecting the properties of the interfaces, so that:

– $\mathscr{F}_+ \cup \mathscr{F}_- \cup (\overline{\mathscr{F}_+} \cap \overline{\mathscr{F}_-}) = \Omega$,

– the phases do not slip one on the other at the interface,
– we have continuity of the normal stresses at the interface,

we have that the extended unknowns

$$\rho = \rho_+ \mathbf{1}_{\mathscr{J}_+} + \rho_- \mathbf{1}_{\mathscr{J}_-}, \quad u = u_+ \mathbf{1}_{\mathscr{J}_+} + u_- \mathbf{1}_{\mathscr{J}_-},$$

satisfy the compressible Navier Stokes equations on the whole container Ω :

$$\partial_t \rho + \operatorname{div}(\rho u) = 0, \tag{1}$$

$$\partial_t(\rho u) + \operatorname{div}(\rho u \otimes u) = \operatorname{div}\Sigma. \tag{2}$$

Assuming further that the densities of the different phases range two non-overlapping closed intervals I_+ and I_-, we can complement this system by the equations of states:

$$\Sigma_i = 2\mu(\nabla u + \nabla^\top u) + (l \operatorname{div} u - p)\mathbb{I}_3, \tag{3}$$

$$p = p(\rho), \quad \mu = \mu(\rho), \quad l = l(\rho), \tag{4}$$

where the functions p, μ, l extend the values for the two components:

$$p(\rho) = p_i(\rho), \quad \mu(\rho) = \mu_i, \quad l(\rho) = \lambda_i, \quad \forall\, \rho \in I_i, \qquad i = +, -.$$

In this paper, we restrict to the one-space dimension setting namely:

$$\partial_t \rho + \partial_x(\rho u) = 0, \tag{5}$$

$$\partial_t \rho u + \partial_x(\rho u^2) = \partial_x(\mu(\rho)\partial_x u) - \partial_x p, \tag{6}$$

$$p = p(\rho), \quad \mu = \mu(\rho). \tag{7}$$

We aim to compute a homogenized system for configurations in which any time/space cell of arbitrary small size contains a fraction of phase + and a fraction of phase -. In the bifluid setting, a possible method is to look for two-scale solutions (a kind of WKB expansion) of the following form:

$$\rho(t,x) = \sum_{i=+,-} \theta_i\left(t, \frac{t}{\varepsilon}, x, \frac{x}{\varepsilon}\right) \rho_i^\varepsilon(t,x), \tag{8}$$

$$u(t,x) = u_0\left(t, \frac{t}{\varepsilon}, x, \frac{x}{\varepsilon}\right) + \varepsilon u_1\left(t, \frac{t}{\varepsilon}, x, \frac{x}{\varepsilon}\right) + \varepsilon^2 u_2\left(t, \frac{t}{\varepsilon}, x, \frac{x}{\varepsilon}\right) + O(\varepsilon^3)\,. \tag{9}$$

Assuming further that

$$\rho_i^\varepsilon(t,x) = \rho_i^0(t,x) + O(\varepsilon), \quad \theta_i(t,\tau,x,y) \in \{0,1\} \quad \text{a.e.}, \tag{10}$$