

EUCLID'S *ELEMENTS* IN LATIN, 1482–1703: VOCABULARY AND CLASSIFICATION

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Abstract. — This paper attempts an improved classification of the Latin texts of the *Elements* printed between 1482 and 1703. It uses primarily a comparison of the 114 terms defined within the text, supplemented by a comparison of the number of definitions included in each book and spot-check comparison of portions of the text and diagrams. The result is a classification into fourteen families. The process of classification permits some reflections on the degree to which the Latin Euclidean vocabulary stabilized during this period, and on the practices of the editors of these versions of the text, which prove to have been in a high proportion of cases eclectic, with definitions, enunciations, proofs and diagrams frequently based on different sources or modified idiosyncratically, even in cases where a specific model was named in the paratext.

Résumé. — Cet article propose une classification améliorée des textes latins des *Éléments* imprimés entre 1482 et 1703. Il utilise principalement une comparaison des 114 termes définis dans le texte, complétée par une comparaison du nombre de définitions incluses dans chaque livre et une vérification ponctuelle de certaines parties du texte et des diagrammes. Le résultat est une classification en quatorze familles. Le processus de classification permet quelques réflexions sur la mesure dans laquelle le vocabulaire latin euclidien s'est stabilisé durant cette période, ainsi que sur les pratiques des éditeurs de ces versions du texte, qui se sont avérées être dans une grande proportion de cas éclectiques, avec des définitions, des énoncés, des preuves et des diagrammes souvent basés sur différentes sources

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1. INTRODUCTION

The Latin tradition of the Euclidean *Elements* in the early modern period presents the appearance of a forbidding complexity, with more than a hundred different impressions whose interrelationships are seldom clear or explicit. Anyone who has worked with these texts will be familiar with the sense of eclecticism that many of them project, but it has not so far been possible to give more than an impressionistic account of the shape of the tradition. In some cases interrelationships are stated or suggested by the paratext, or within the main text, but more often it is at least partially unclear on what sources a particular version of the text was based. There are certainly cases in which a paratext is misleading in these respects. Because the number of texts is large and the degree of variability is high, full collation is both infeasible and likely to be ineffective as a means of characterising the interrelationships of different texts or dividing them into families.

This paper presents a new classification of the Latin versions of the *Elements* from this period, based primarily on their vocabulary choices. It examines specifically the period 1482–1703, that is from the *editio princeps* of Ratdolt to the relatively scholarly edition by Gregory. It comments on the degree of stability of different parts of that vocabulary, as well as the rate at which it stabilised over the period, the degree to which different individual editors innovated or conformed with respect to their predecessors, and the degree of uptake which their innovations (if any) enjoyed. It combines this information with the variation in proposition lists from one edition to another, and with limited collation of portions of the texts themselves, in order to attempt a classification and stemma. It explicitly aims to improve on the classificatory work presented in Wardhaugh et al. [2024], which was based only on the criterion of proposition counts. The data from that catalogue have also been imported into the *Medée* catalogue of Euclidean texts Herreman & Vitrac [n.d.], which provides a fine-grained, network-type presentation in which items are handled case-by-case, and with a relatively wide criterion for inclusion. Herreman and Vitrac also provide an important discussion of the typology of dependencies between Euclidean texts. What is attempted in the present paper is intended as complementary to the approach of the *Medée* project, using

precise, replicable criteria to arrive at a relatively compact characterisation of the different families of texts, and is limited to a more uniform corpus, namely printed Latin texts which were in intention editions of some part of the Euclidean text. The ultimate aim of this work is to understand better the practices of early modern Euclidean editors, and by extension of mathematical editors in this period more generally.

2. METHOD

2.1. *Texts*

A list of Latin editions of the *Elements* was compiled based on the catalogue Wardhaugh et al. [2024]; included were all Latin versions of the text which are not described there as re-impressions or re-issues of a specific predecessor. Most are texts in which a given editor is named for the first time, but second or subsequent editions by the same editor are included where there is reason to believe that substantial modifications were made, such as in the important second editions of Foix, Clavius and Dechales. A total of sixty texts were included; a complete list appears in Appendix 1. In two cases it was necessary to substitute a more accessible later reimpression of (what is believed to be) substantially the same version: namely Tacquet (1665), substituting for the original impression of 1654, and Dechales (1675), in place of the 1660 original. One version, Porpora (1702), was excluded from the main cycle of comparison due to the author having no access to either a copy or a complete surrogate; in fact it appears to be a reimpression of a specific predecessor, as mentioned below (section 3.4/Commandino). Another, Maurolico (1575), was excluded from the comparison of vocabulary as containing none of the books in which the terms collated are defined; its definition counts and text were examined in the other ways described below.

2.2. *Geometrical terms*

Across these versions of the *Elements*, this study collated 114 vocabulary items introduced in books I–VII, X and XI. The list of terms follows that of Wardhaugh [2024a]: it includes all terms introduced in a portion of the text labelled a ‘definition’, as well as certain terms defined in the enunciations to propositions, namely those for classes of irrational line in book X. (It excludes structural terms such as ‘proposition’, ‘theorem’ and so on.) Duplicate appearances of the same term are generally avoided: ‘even’ and ‘odd’ (def. 7.6, 7.7) are included, for instance, but not ‘evenly

even' and 'evenly odd' (def. 7.8–7.10), definitions in which no new term is introduced. Terms defined more than once, such as 'centre' or 'cube' are included only once. Attention was paid, however, to the fact that in some cases the same Greek term is rendered differently in different contexts by some editors: *πέρατα* is the most important example, defined twice in book I and once in book XI (def. 1.3, 1.6, 11.2), sometimes translated using two or three different words (as in English: *ends, edges, faces*). All three appearances are included. A few additional terms are included, commonly defined in the early modern texts though not in the Greek text established by Heiberg (Heiberg/Stamatēs 1969–77): *ordinata proportio* in book V, ἀντιπεπονηώς [reciprocally related] and πελικότης ['size' of a ratio] in book VI, and τετράεδρον [tetrahedron] and στερεός παραλληλεπίπεδος [parallelepipedal solid] in book XI. A full list of the terms collated appears in Appendix 2. The numbering of definitions varies widely from one version of the text to others (and in some it is absent); in what follows, definition numbers are those of the Greek text of Heiberg.

The vocabulary items show superficially a complex pattern, with minor differences certainly the norm even between editions known to be closely related. It was evident, however, that a richer picture of relationships between editions could be derived from this set of data. After initial collation and checking, a normalized version of the vocabulary lists was prepared, with the intention of discarding (a) differences of spelling, punctuation, capitalization or abbreviation; (b) differences in the number and case of nouns and adjectives, or in the person, number, tense and voice of verbs resulting from changes of phrasing. The first of these criteria is easily applied; the second involves the exercise of some judgement. Zamberti's innovation *dimetiens*, for instance, was included as a distinct choice, not conflated with *diameter*, despite being arguably no more than a different expression of the same verbal root. Similarly *ex aequalitate*, *ex aequo* and *ex aequali* were taken to be distinct renderings of δι' ἴσου (def. 5.17); and *duplīcata*, *dupla*, *duplo maiorem* and *duplex* of διπλάσιος (5.9).

Inevitably some information about the relationships between editions was lost during this process, since it is quite possible that an idiosyncratic choice of spelling or the decision to word a particular definition in the plural or the passive was in fact a mark of relationship between two versions of the text. The intention, however, was deliberately to achieve the highest possible degree of agreement between texts, so as to make salient those cases where editors differed as to the actual lexemes to be employed.

Once the terms had been normalized, innovative vocabulary choices were identified in each edition and distinct lexemes were counted for

each Euclidean term. This made it possible to interrogate the data in a preliminary way, calculating the frequency with which each individual editor introduced novel vocabulary items, and the frequency with which each individual Greek term received a novel Latin rendering.

In pursuit of the main purpose of this paper—the classification of editions—for each pair of texts the degree of vocabulary agreement was calculated: specifically, the proportion of the later text's (normalized) terms that agreed with those of the earlier text. (This was carried out automatically using a custom VBA macro within Microsoft Excel.) To the resulting grid, listing the degrees of vocabulary agreement for 1,596 pairs of Euclidean texts, various forms of conditional colouring were successively applied in order to render salient the features of the data that were of interest.

This exercise made it clear that some editions elicited vocabulary agreement from their successors frequently, but others hardly at all; and similarly that some editions were in agreement with a high proportion of their predecessors, others with rather fewer. Both of these observations were quantified, by averaging the calculated degrees of agreement respectively across the successors and the predecessors of each edition.

Next, after some experimentation, a criterion of 95% agreement was chosen to distinguish the closely related from the less closely. For each edition, those successors matching this well or better were listed, and this information forms the basis of the discussion of edition families in Section 3 below. A new table of vocabulary items was prepared, with the texts arranged into these families. For each term, it was determined (again using a custom macro) whether or not it varied within each family; for each family the number of terms showing variation was determined. Terms diverging from the norm represented by the 'head' member of each family were highlighted.

2.3. *Propositions*

This study also recorded, for each text, the number of propositions included in each Euclidean book (including the spurious books XIV and XV, and the additional books XVI–XVIII published by Foix in 1566 and 1578). Certain cases described in their paratexts—or obviously intended—as abridgements, such as the selections from books XI and XII in Grienberger (1629), were excluded. These data were added to the table of vocabulary items as arranged into families of editions, and handled in a similar way: for each book it was recorded whether its length showed any