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LOCALLY MOVING GROUPS AND LAMINAR ACTIONS ON THE LINE

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by

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Abstract. – We prove various results that, given a sufficiently rich subgroup G of the group of homeomorphisms on the real line, describe the structure of the other possible actions of G on the line, and address under which conditions such actions must be semi-conjugate to the natural defining action of G. The main assumption is that G should be locally moving, meaning that for every open interval the subgroup of elements fixing pointwise its complement, acts on it without fixed points. One example (among many others) is given by Thompson's group F.

In Part I, we show that when G is a locally moving group, every C^1 action of G on the real line without fixed points is semi-conjugate to its standard action or to a non-faithful action. It turns out that the situation is much wilder when considering actions by homeomorphisms: for a large class of groups, including Thompson's group F, we describe uncountably many conjugacy classes of faithful minimal actions by homeomorphisms on the real line.

In Part II, we prove structure theorems describing the dynamics of exotic C^0 actions, based on the study of laminar actions, which are actions on the line preserving a lamination. When G is a group of homeomorphisms of the line acting minimally, and with a non-trivial compactly supported element, then any faithful minimal action of G on the line is either laminar or conjugate to its standard action. Moreover, when G is a locally moving group satisfying a suitable finite generation condition, we prove that for any faithful minimal laminar action on the line, there is a map from the lamination to the line, called a horograding, which is equivariant with respect to the action on the lamination and the standard action, and satisfies some extra suitable conditions. This establishes a tight relation between all minimal actions on the line of such groups, and their standard actions.

Among the various applications of this result, we show in Part III that for a large class of locally moving groups, the standard action is locally rigid, in the sense