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ALGEBRAIC CYCLES AND HITCHIN SYSTEMS

BY DAVESH MAULIK, JUNLIANG SHEN AND QIZHENG YIN

ABSTRACT. – The purpose of this paper is to study motivic aspects of the Hitchin system for GL_n. Our results include the following. (a) We prove the motivic decomposition conjecture of Corti-Hanamura for the Hitchin system; in particular, the decomposition theorem associated with the Hitchin system is induced by algebraic cycles. This yields an unconditional construction of the motivic perverse filtration for the Hitchin system, which lifts the cohomological/sheaf-theoretic perverse filtration. (b) We prove that the inverse of the relative hard Lefschetz symmetry is induced by a relative algebraic correspondence, confirming the relative Lefschetz standard conjecture for the Hitchin system. (c) We show a strong perversity bound for the normalized Chern classes of a universal bundle with respect to the motivic perverse filtration; this specializes to the sheaf-theoretic result obtained earlier by Maulik-Shen. (d) We prove a χ-independence result for the relative Chow motives associated with Hitchin systems.

Our methods combine Fourier transforms for compactified Jacobian fibrations associated with integral locally planar curves, nearby and vanishing cycle techniques, and a Springer-theoretic interpretation of parabolic Hitchin moduli spaces.

RÉSUMÉ. – L'objectif de cet article est d'étudier des aspects motiviques du système de Hitchin pour GL_n. Nos résultats incluent les suivants. (a) Nous prouvons la conjecture de décomposition motivique de Corti-Hanamura pour le système de Hitchin ; en particulier, le théorème de décomposition associé au système de Hitchin est induit par des cycles algébriques. Cela fournit une construction inconditionnelle de la filtration perverse motivique pour le système de Hitchin, qui relève la filtration perverse cohomologique/faisceautique. (b) Nous prouvons que l'inverse de la symétrie de Lefschetz difficile relative est induit par une correspondance algébrique relative, ce qui confirme la conjecture standard de Lefschetz relative pour le système de Hitchin. (c) Nous montrons une borne de perversité forte pour les classes de Chern normalisées d'un fibré universel par rapport à la filtration perverse motivique ; cela se restreint au résultat faisceautique obtenu précédemment par Maulik-Shen. (d) Nous prouvons un résultat de χ-indépendance pour les motifs de Chow relatifs associés aux systèmes de Hitchin.

Nos méthodes combinent les transformées de Fourier pour les fibrations jacobiennes compactifiées associées aux courbes intégrales localement planes, des techniques de cycles proches et évanescents, et une interprétation en termes de la théorie de Springer des espaces de modules de Hitchin paraboliques.

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1. Introduction

The topology of the Hitchin system has been intensively studied over decades. In [54], Ngô proved the fundamental lemma of the Langlands program by reducing it to the study of the decomposition theorem [8] for the Hitchin system; the topological mirror symmetry conjecture of Hausel-Thaddeus [32] was proven by comparing the decomposition theorem for Hitchin systems associated with the Langlands dual groups SL_n and PGL_n [29, 44]; the decomposition theorem for the Hitchin system further yields a perverse filtration on the singular cohomology of the Hitchin moduli space, whose interaction with the non-abelian Hodge theory is the main theme of the P=W conjecture [12], recently proven [46, 30, 47]; the decomposition theorem for the Hitchin system also appears naturally in enumerative geometry, particularly in the study of BPS invariants [18, 45, 36, 20].

Most of these developments relied heavily on topological and sheaf-theoretic approaches. Nevertheless, hints from several recent results [28, 41, 35, 47] suggest that there should be a motivic theory hidden behind the cohomological structures mentioned above. The purpose of this work is to study motivic aspects of the decomposition theorem and the perverse filtration for the Hitchin system. The upshot is that the geometry of the Hitchin system provides us powerful tools to construct algebraic cycles on the moduli of Higgs bundles, lifting many cohomological statements to the motivic level.

1.1. Hitchin systems

Throughout, we work over the complex numbers $\mathbb{C}$; all Chow groups are taken with rational coefficients.

Let Σ be a nonsingular projective curve of genus $g \geq 2$. Let $M_{n,d}$ be the moduli space of stable Higgs bundles with coprime rank n and degree d on Σ; it carries the structure of an integrable system, known as the *Hitchin system*:

$$f : M_{n,d} \to B, \tag{1}$$

which is a proper and surjective morphism to an affine space B. We use d_f to denote the relative dimension of (1). By the decomposition theorem of Beilinson, Bernstein, Deligne, and Gabber [8], the derived pushforward $Rf_*\mathbb{Q}_{M_{n,d}} \in D^{\mathrm{b}}_{\mathrm{c}}(B)$ admits a decomposition into (shifted) semisimple perverse sheaves:

$$Rf_*\mathbb{Q}_{M_{n,d}}[\dim M_{n,d} - d_f] \simeq \bigoplus_{i=0}^{2d_f} {}^{\mathfrak{p}}\mathcal{H}^i\left(Rf_*\mathbb{Q}_{M_{n,d}}[\dim M_{n,d} - d_f]\right)[-i] \in D^{\mathrm{b}}_{\mathrm{c}}(B), \tag{2}$$

where the semisimple summands on the right-hand side satisfy the Lefschetz symmetry

$$\sigma^i : {}^{\mathfrak{p}}\mathcal{H}^{d_f-i}\left(Rf_*\mathbb{Q}_{M_{n,d}}[\dim M_{n,d} - d_f]\right) \xrightarrow{\simeq} {}^{\mathfrak{p}}\mathcal{H}^{d_f+i}\left(Rf_*\mathbb{Q}_{M_{n,d}}[\dim M_{n,d} - d_f]\right) \tag{3}$$

induced by the cup-product with the i-th power of a relative ample class σ. The decomposition (2) is an important invariant for the Hitchin system (1) and has played a crucial role in all the stories mentioned above. For example, recent resolutions of the P=W conjecture show that the (cohomological) perverse filtration

$$P_0H^*(M_{n,d}, \mathbb{Q}) \subset P_1H^*(M_{n,d}, \mathbb{Q}) \subset \cdots \subset H^*(M_{n,d}, \mathbb{Q}) \tag{4}$$

induced by (2) matches the double-indexed weight filtration associated with the corresponding character variety via non-abelian Hodge theory:

$$P_k H^*(M_{n,d}, \mathbb{Q}) = W_{2k} H^*(M_{\mathrm{char}}, \mathbb{Q}).$$

Furthermore, the Lefschetz symmetry (3) matches the *curious hard Lefschetz* for the character variety [31, 49].

Our first result is that both the decomposition (2) and the filtration (4) are motivic in a strong sense. In particular, this confirms the *motivic decomposition conjecture* of Corti-Hanamura [19] for the Hitchin system.

THEOREM 1.1 (Motivic decomposition). – *There exists a decomposition of the relative diagonal into orthogonal projectors:*

$$[\Delta_{M_{n,d}/B}] = \sum_{i=0}^{2d_f} \mathfrak{r}_i, \qquad \mathfrak{r}_i \circ \mathfrak{r}_i = \mathfrak{r}_i, \quad \mathfrak{r}_i \circ \mathfrak{r}_j = 0, \quad i \neq j,$$

which induces a sheaf-theoretic decomposition (2)*. Here* $[\Delta_{M_{n,d}/B}]$ *and* $\mathfrak{r}_i$ *lie in the Chow group of the relative product* $M_{n,d} \times_B M_{n,d}$*, and the composition is taken as relative correspondences.*

Our method also yields immediately the relative Lefschetz standard conjecture for the Hitchin system, which shows the algebraicity of the inverse of (3).

THEOREM 1.2 (Relative Lefschetz). – *For any relative ample class* σ *and* $i > 0$*, the inverse of the Lefschetz symmetry* (3) *is induced by an algebraic cycle* $\mathfrak{Z}_{\sigma,i}$ *on* $M_{n,d} \times_B M_{n,d}$*:*

$$\mathfrak{Z}_{\sigma,i} : {}^{\mathfrak{p}}\mathcal{H}^{d_f+i}\left(Rf_*\mathbb{Q}_{M_{n,d}}[\dim M_{n,d} - d_f]\right) \xrightarrow{\simeq} {}^{\mathfrak{p}}\mathcal{H}^{d_f-i}\left(Rf_*\mathbb{Q}_{M_{n,d}}[\dim M_{n,d} - d_f]\right).$$

Under the language of [19], we obtain from Theorem 1.1 a decomposition in the category of relative Chow motives over B:

$$h\left(M_{n,d}\right) = \bigoplus_i h_i(M_{n,d}) \in \mathrm{CHM}(B) \tag{5}$$

with

$$h(M_{n,d}) = (M_{n,d}, [\Delta_{M_{n,d}/B}], 0), \quad h_i(M_{n,d}) = (M_{n,d}, \mathfrak{r}_i, 0)$$

whose homological realization under the Corti-Hanamura functor

$$\mathrm{CHM}(B) \to D^{\mathrm{b}}_{\mathrm{c}}(B)$$

recovers (2). As a consequence, we obtain a motivic perverse filtration

$$P_\bullet h(M_{n,d}) \subset h(M_{n,d}); \tag{6}$$

see Section 3.1. This motivic perverse filtration specializes to both the cohomological perverse filtration (4) and a perverse filtration on the Chow groups of $M_{n,d}$; the latter has not yet been much studied.

REMARK 1.3. – We conclude Section 1.1 with some remarks.

(a) The decomposition (5) we constructed in Theorem 1.1 specializes to a particular sheaf-theoretic isomorphism (2). Conversely, it is interesting to explore whether the natural sheaf-theoretic decompositions [21, 10] admit motivic liftings.