

Astérisque

HENRY C. PINKHAM

Deformations of algebraic varieties with G_m action

Astérisque, tome 20 (1974)

<http://www.numdam.org/item?id=AST_1974__20__1_0>

© Société mathématique de France, 1974, tous droits réservés.

L'accès aux archives de la collection « Astérisque » (<http://smf4.emath.fr/Publications/Asterisque/>) implique l'accord avec les conditions générales d'utilisation (<http://www.numdam.org/conditions>). Toute utilisation commerciale ou impression systématique est constitutive d'une infraction pénale. Toute copie ou impression de ce fichier doit contenir la présente mention de copyright.

NUMDAM

Article numérisé dans le cadre du programme
Numérisation de documents anciens mathématiques
<http://www.numdam.org/>

Table of Contents

		Page No.
	1. Introduction	3
Chapter I.	Generalities on deformations of varieties with $\mathbb{G}_m$ action	16
	2. Deformations of singularities with $\mathbb{G}_m$ action	16
	3. A sketch of Schlessinger's results for cones over projectively normal varieties	23
	4. A theorem on negatively graded cones	27
	5. The theorem on negative grading when X is projectively normal	29
Chapter II.	Deformations of cones over projectively normal curves	36
	6. Mumford's study of T^1	36
	7. Smooth deformations of cones over curves	42
	8. The cone over the rational curve of degree n in $\mathbb{P}^n$	48
	9. Cones over elliptic curves	63
Chapter III.	One-dimensional cones	74
	10. Computation of T^1 for Gorenstein curves	74
	11. Deformations of cones of lines	79

Chapter IV.	Monomial curves	92
12.	Monomial curves: definitions and study of T^1	92
13.	Monomial curves and Weierstrass points	99
14.	The existence of smooth deformations of monomial curves in certain special cases.	108
15.	An example	121
Bibliography		127

1. Introduction

This thesis^{*} is devoted to a study of the deformations of an isolated singularity of an algebraic variety admitting a multiplicative group action. In this section we give a summary of the techniques used and the results obtained.

(1.1) First we set up some notation and review the relevant elements of deformation theory. Details can be found in [43] and [45]. We fix once and for all an algebraically closed field k .

Let B be a local k -algebra of finite type. Let $\mathcal{C}$ be the category of local artin k -algebras, $\hat{\mathcal{C}}$ that of complete noetherian local k -algebras: hence if $A \in \hat{\mathcal{C}}$ with maximal ideal m , then $A/m^i \in \mathcal{C}$ for all i .

Definition (1.2) An infinitesimal deformation of B to $A \in \mathcal{C}$ is a cartesian diagram

$$\begin{array}{ccc} B' & \xrightarrow{\quad} & B = B' \otimes_A k \\ \uparrow & & \uparrow \\ A & \xrightarrow{\text{res}} & k \end{array}$$

where $A \longrightarrow B'$ is flat.

* A slightly different version of this work was submitted to Harvard University in partial fulfillment of the Ph.D. requirements in May 1974.

Definition (1.3) D , the local deformation functor from $\mathcal{C}$ to {sets} takes $A \in \mathcal{C}$ to isomorphism classes of deformations of B to A , isomorphism being defined in the obvious way. We extend D to $\hat{\mathcal{C}}$ by taking inverse limits to get the notion of formal deformation.

We will usually be interested in minimal "complete" families of formal deformations, in the following sense:

Definition (1.4) A formal deformation $\zeta \in D(R)$, $R \in \hat{\mathcal{C}}$ is versal if

(i) any formal deformation $\zeta' \in D(S)$ may be deduced from ζ by a base change $R \longrightarrow S$.

(ii) the Zariski tangent space of R is minimal for (i). R (or $\text{Spec } R$) is then called the formal moduli space of B ; its tangent space (the k -vector space of first order deformations) is denoted T_B^1 , or just T^1 if no confusion is possible.

Lichtenbaum-Schlessinger [28] have studied T^1 using the cotangent complex. Schlessinger's theorem [43] implies that when $\text{Spec } B$ has an isolated singularity at its closed point, then B has a versal deformation.