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Astérisque, tome 310 (2006)

<http://www.numdam.org/item?id=AST_2006__310__R1_0>

ASTÉRISQUE 310

SELMER COMPLEXES

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Société Mathématique de France 2006
Publié avec le concours du Centre National de la Recherche Scientifique

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2000 *Mathematics Subject Classification.* — 11R23, 11G40, 11F41.

Key words and phrases. — Iwasawa theory, Selmer groups, big Galois representations.

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Abstract. — This book builds new foundations of Iwasawa theory, based on a systematic study of cohomological invariants of big Galois representations in the framework of derived categories. A new duality formalism is developed, which leads to generalized Cassels-Tate pairings and generalized p-adic height pairings. One of the applications is a parity result for Selmer groups associated to Hilbert modular forms.

Résumé **(Complexes de Selmer).** — Ce livre construit de nouvelles fondations pour la théorie d'Iwasawa, basées sur une étude systématique d'invariants cohomologiques (vivant dans des catégories dérivées) pour les grosses représentations galoisiennes. On développe un nouveau formalisme de dualité dont on déduit des accouplements de Cassels-Tate généralisés et des hauteurs p-adiques généralisées. Une des applications est un résultat de parité pour les groupes de Selmer attachés aux formes modulaires de Hilbert.

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CHAPTER 0

INTRODUCTION

0.0. Big Galois representations

In this work we study cohomological invariants of "big Galois representations"

$$\rho : G \longrightarrow \mathrm{Aut}_R(T),$$

where

(i) G is a suitable Galois group.

(ii) R is a complete local Noetherian ring, with a finite residue field of characteristic p.

(iii) T is an R-module of finite type.

(iv) ρ is a continuous homomorphism of pro-finite groups.

We develop a general machinery that covers duality theory, Iwasawa theory, generalized Cassels-Tate pairings and generalized height pairings.

0.1. Examples

0.1.0. An archetypal example of a big Galois representation arises as follows. Let K be a field of characteristic $\mathrm{char}(K) \neq p$. For every K-scheme $X \to \mathrm{Spec}(K)$ put $\overline{X} = X \otimes_K K^{\mathrm{sep}}$, where K^{sep} is a fixed separable closure of K. Given a projective system $X_\infty = (X_\alpha)_{\alpha \in I}$ (indexed by some directed set I) of separated K-schemes of finite type with *finite* transition morphisms $X_\beta \to X_\alpha$, put

$$H^i(X_\infty) := \varprojlim_{\alpha} H^i_{\mathrm{et}}(\overline{X}_\alpha, \mathbf{Z}_p) = \varprojlim_{\alpha} \varprojlim_{n} H^i_{\mathrm{et}}(\overline{X}_\alpha, \mathbf{Z}/p^n\mathbf{Z}),$$

where the transition morphisms are given by trace maps. This is a representation of $G_K = \mathrm{Gal}(K^{\mathrm{sep}}/K)$, linear over the $\mathbf{Z}_p$-algebra generated by "endomorphisms" of the tower X_∞. In practice, $H^i(X_\infty)$ is often too big and must be first decomposed into smaller constituents. One can also use more general coefficient sheaves, not just $\mathbf{Z}/p^n\mathbf{Z}$.