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**FROM HOMOTOPY LINKS
TO STRATIFIED
HOMOTOPY THEORIES**

S. DOUTEAU & L. WAAS

**FLUID DYNAMIC LIMIT
OF BOLTZMANN EQUATION
FOR GRANULAR HARD-SPHERES
IN A NEARLY ELASTIC REGIME**

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