Revue d'Histoire des Mathématiques

The Kaṭapayādi *system of numerical notation*

Sreeramula Rajeswara Sarma

Tome 18 Fascicule 2 2012

SOCIÉTÉ MATHÉMATIQUE DE FRANCE

Publiée avec le concours du Centre national de la recherche scientifique

REVUE D'HISTOIRE DES MATHÉMATIQUES

RÉDACTION

Rédacteur en chef :
Norbert Schappacher

Rédacteur en chef adjoint :
Frédéric Brechenmacher

Membres du Comité de rédaction :
Maarten Bullynck
Sébastien Gandon
Veronica Gavagna
Hélène Gispert
Catherine Goldstein
Marc Moyon
Karen Parshall
Silvia Roero
Tatiana Roque
Ivahn Smadja

Directeur de la publication :
Stéphane Seuret

COMITÉ DE LECTURE

Philippe Abgrall
June Barrow-Green
Umberto Bottazzini
Jean Pierre Bourguignon
Aldo Brigaglia
Bernard Bru
Jean-Luc Chabert
François Charette
Karine Chemla
Pierre Crépel
François De Gandt
Moritz Epple
Natalia Ermolaëva
Christian Gilain
Jeremy Gray
Tinne Hoff Kjeldsen
Jesper Lützen
Antoni Malet
Irène Passeron
Jeanne Peiffer
Christine Proust
Sophie Roux
David Rowe
Ken Saito
S. R. Sarma
Erhard Scholz
Reinhard Siegmund-Schultze
Stephen Stigler
Bernard Vitrac

Secrétariat :
Nathalie Christiaën
Société Mathématique de France
Institut Henri Poincaré
11, rue Pierre et Marie Curie, 75231 Paris Cedex 05
Tél. : (33) 01 44 27 67 99 / Fax : (33) 01 40 46 90 96
Mél : rhmsmf@ihp.fr / URL : http//smf.emath.fr/

Périodicité : La *Revue* publie deux fascicules par an, de 150 pages chacun environ.

Tarifs : Prix public Europe : 89 €; prix public hors Europe : 97 €;
prix au numéro : 43 €.
Des conditions spéciales sont accordées aux membres de la SMF.

Diffusion : SMF, Maison de la SMF, Case 916 - Luminy, 13288 Marseille Cedex 9
Hindustan Book Agency, O-131, The Shopping Mall, Arjun Marg, DLF Phase 1, Gurgaon 122002, Haryana, Inde

© SMF

Nº ISSN : 1262-022X

Maquette couverture : Armelle Stosskopf

Revue d'histoire des mathématiques
18 (2012), p. 37–66

THE *KAṬAPAYĀDI* SYSTEM OF NUMERICAL NOTATION AND ITS SPREAD OUTSIDE KERALA

Sreeramula Rajeswara Sarma

Abstract. — While the study of the transmission of scientific ideas from and to India has its own importance, it is also necessary to examine the transmission of ideas within India, from one region to another, from Sanskrit to regional languages and vice versa. This paper attempts to map the spread of the *Kaṭapayādi* system of numerical notation, widely popular in Kerala, to other parts of India, and shows that this very useful tool of mathematical notation, though well known in northern India, was rarely employed there.

The paper further refutes the contention of Bibhutibhusan Datta and Avadhesh Narayan Singh that there existed four distinct variants of the *Kaṭapayādi* system and shows that there were only two genuine variants of the system, one of these being limited to just one single text of unknown provenance.

Résumé (Le système de notation numérique *Kaṭapayādi* et sa diffusion en dehors du Kérala)

Alors que l'étude de la transmission des idées scientifiques entre l'Inde et le reste du monde a sa propre importance, il est également nécessaire d'examiner la transmission de celles-ci à l'intérieur du pays, entre les régions, de la langue sanskrite vers les langues régionales et vice-versa. Cet article se propose de cartographier la diffusion du système *Kaṭapayādi* de notation numérique, largement répandu au Kérala, aux others parties de l'Inde, et montre que ce très important outil de notation mathématique, alors que bien connu en Inde du Nord, y a été rarement employé.

D'autre part, cet article refute l'affirmation de Bibhutibhusan Datta et d'Avadhesh Narayan Singh sur l'existence de quatre variantes distinctes du

Texte reçu le 9 décembre 2010, accepté pour publication le 27 mai 2011, texte définitif reçu le 1er septembre 2011.

S. R. Sarma, Höhenstraße 28, 40227 Düsseldorf, Germany.

Courrier électronique : sr@sarma.de

Key words and phrases : *Akṣarasaṃkhyā*, Bhālūmal, Bhāskara II, *Bhūtasaṃkhyā*, Harṣa, inscriptions, *Kaṭapayādi* system, magic squares, Rāmacandra Vājapeyin, Sanskrit astrolabes, Sanskrit celestial globes.

© Société mathématique de France, 2012

système *Kaṭapayādi* et montre qu'en réalité il n'y en avait que deux, l'une d'entre elles étant limitée à un seul texte, d'origine inconnue.

INTRODUCTION

Indian astronomers/mathematicians developed two major strategies [1] for expressing large numbers consisting of several digits in the metrical pattern of Sanskrit verses with which they composed their works. In the first method, commonly known as *Bhūtasaṃkhyā* or word numerals, the digits 1 to 9 and zero were expressed by certain significant words. [2] Thus, for example, "moon" stands for 1 as there is only one moon; words like "eye" or "hand" represent 2, because "eyes" or "hands" occur always in pairs. In the second system called *Kaṭapayādi*, the digits are represented by the consonants of the Sanskrit alphabet. For example, 1 is denoted by *k*, *ṭ*, *p*, or *y*. Both the methods operate in the decimal place value system, and in both cases, the enumeration commences with the units' place and proceeds to the next higher powers of ten.

This may be illustrated as follows. In the *Bhūtasaṃkhyā* system, the year 2010 can be expressed as *ākāśa-vasudhā-nabhas-kara*. Here *ākāśa* = sky = 0, *vasudhā* = earth = 1, *nabhas* = sky = 0, *kara* = hand = 2. Since the enumeration begins with the units' place, the order of the digits has to be reversed, and the numerical equivalent of *ākāśa-vasudhā-nabhas-kara* is 2010. With its great wealth of synonyms, Sanskrit is capable of expressing numbers in many ways according to the exigencies of the verse meter; thus 2010 can also be expressed as *ananta-soma-kha-hasta* or *viyac-candra-gagana-nayana* and so on.

In the *Kaṭapayādi* system, 2010 can be denoted by *naṭanara* (*na* = 0, *ṭa* = 1, *na* = 0, *ra* = 2; reading backwards 2010). Here the consonants are chosen in such a way that their combination has some meaning and is not a mere jumble of sounds; *naṭanara* means "actor-man" or "a man who is an actor". Since three or four consonants are available for each digit (there are, however, only two consonants for the zero), and since any vowel can be added to a consonant, a number can be expressed in a variety of ways.

---

1 In his *Āryabhaṭīya*, Āryabhaṭa I employs an alphabetical notation, which is distinct from the *Bhūtasaṃkhyā* and *Kaṭapayādi*; cf. [Āryabhaṭa I 1976, 1.1, p. 3–5].

2 This system provides symbolic words for a few two-digit numbers as well, such as *dik* (10), *rudra* (11), *tithi* (15), and so on; cf. [Sarma 2003, p. 61–62].

Of these two systems, the *Bhūtasaṃkhyā* is older [3] and was used throughout India and also in South-East Asia, while the latter is essentially a feature of Kerala. In this paper I propose to survey the current state of research on the *Kaṭapayādi* system and discuss some hitherto unknown cases of its spread outside Kerala.

1. THE *KAṬAPAYĀDI* SYSTEM IN KERALA

The *Kaṭapayādi* system is an alphabetical notation where each consonant of the Sanskrit alphabet is given a numerical value. [4] The system is described in an anonymous line thus: *kādi nava, ṭādi nava, pādi pañca, yādy aṣṭau*, "the nine [consonants] starting with *ka*, the nine starting with *ṭa*, the five starting with *pa* and the eight starting with *ya* [respectively denote the numbers 1 to 9]." But it does not say how the zero is to be represented. The *Sadratnamālā* ("Garland of Pure Gems") by Śaṅkaravarman, composed in 1819 AD, gives a comprehensive definition by stating that "*na, ña* and the vowels [standing alone denote] zeros; [the consonants] beginning with *ka, ṭa, pa* and *ya*, [denote severally] the numbers [1 to 9]; in a conjunct [consonant, only] the consonant in the penultimate [place, which is followed by a vowel, denotes a] number; and a consonant not attached to a vowel should not be considered [as a number]." [5] This may be graphically shown in the following table:

1	2	3	4	5	6	7	8	9	0
ka	*kha*	*ga*	*gha*	*ṅa*	*ca*	*cha*	*ja*	*jha*	*ña*
ṭa	*ṭha*	*ḍa*	*ḍha*	*ṇa*	*ta*	*tha*	*da*	*dha*	*na*
pa	*pha*	*ba*	*bha*	*ma*					
ya	*ra*	*la*	*va*	*śa*	*ṣa*	*sa*	*ha*		unattached vowels

[3] Symbolic words representing numbers are used sporadically in the *Śrauta-sūtras* and *Vedāṅga-jyotiṣa*. Pingala's *Chandaḥsūtra*, which is generally placed in the second century BC, is the earliest text where these are employed rather extensively, with about a hundred occurrences; cf. [Sarma 2009b].

[4] The important literature on the *Kaṭapayādi* system is the following, arranged in chronological order: [Warren 1825, p. 335], [Whish 1827], [Jacquet 1835, p. 122–130], [Ojha 1971, p. 123], [Fleet 1911], [Fleet 1912], [Datta & Singh 1962, Part 1, p. 69–72], [Raja 1963], [Sarma 1972, p. 6–8], [Subbarayappa & Sarma 1985, p. 47–48], [Sarma 2003], [Yano 2006, p. 150–153], [Plofker 2009, p. 75–77].

[5] [Śaṅkaravarman 2001, 3.3, p. 22]:
nañāv acaś ca śūnyāni saṃkhyāḥ kaṭapayādayaḥ |
miśre tūpāntahal saṃkhyā na ca cintyo hal asvaraḥ ||
Though the *Kaṭapayādi* system was followed extensively for a very long time in Kerala, it is strange that there are no definitions earlier than this one in the *Sadratnamālā* of the nineteenth century! Perhaps it was thought unnecessary to define it as it was universally known.