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FLUID DYNAMIC LIMIT OF BOLTZMANN EQUATION FOR GRANULAR HARD-SPHERES IN A NEARLY ELASTIC REGIME

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Dedicated to the memory of Claude Bardos,
with deep admiration.

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Abstract. **–** In this paper, we provide the first rigorous derivation of hydrodynamic equations from the Boltzmann equation for inelastic hard spheres with small inelasticity. The hydrodynamic system that we obtain is an incompressible Navier-Stokes-Fourier system with self-consistent forcing terms and, to our knowledge, it is thus the first hydrodynamic system that properly describes rapid granular flows consistently with the kinetic formulation. To this end, we write our Boltzmann equation in a non dimensional form using the dimensionless Knudsen number which is intended to be sent to 0. There are several difficulties in such derivation, the first one coming from the fact that the original Boltzmann equation is free-cooling and, thus, requires a self-similar change of variables to introduce an homogeneous steady state. Such a homogeneous state is not explicit and is heavy-tailed, which is a major obstacle to adapting energy estimates. Additionally, a central challenge is to understand the relation between the restitution coefficient, which quantifies the energy loss at the microscopic level, and the Knudsen number. This is achieved by identifying the correct *nearly elastic regime* to capture nontrivial hydrodynamic behavior. We are, then, able to prove exponential stability uniformly with respect to the Knudsen number for solutions of the rescaled Boltzmann equation in a close to equilibrium regime. Finally, we prove that solutions to the Boltzmann equation converge in a specific weak sense towards a hydrodynamic limit which depends on time and space variables only through macroscopic quantities. Such macroscopic quantities are solutions to a suitable modification of the incompressible Navier-Stokes-Fourier system which appears to be new in this context.