TEICHMÜLLER THEORY AND DYNAMICS

Pierre Dehornoy & Erwan Lanneau (eds.)



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***Abstract.* —** This edition of *Panoramas & Synthèses* follows the 27th edition of the summer School in mathematics, focussed on *Teichmüller dynamics, mapping class groups and applications.* It took place from 11 to 22 June 2018 at the Institut Fourier (UMR CNRS 5582) of Grenoble. During this school, twelve specialists came to present the basics of the theory of translation surfaces and their moduli spaces, as well as the recent advances in the field. This volume brings together four texts, all based on the lecture notes of the school, and illustrates the interaction between Teichmüller theory and dynamics.

***Résumé.* (Théorie de Teichmüller et dynamique) —** Ce volume de *Panoramas & Synthèses* fait suite à la 27$^{\text{e}}$ édition de l'école d'été de mathématiques qui portait sur le thème *Teichmüller dynamics, mapping class groups and applications.* Elle s'est déroulée du 11 au 22 juin 2018 à l'Institut Fourier (UMR CNRS 5582) de Grenoble. Lors de cette école, douze spécialistes sont venus présenter les bases de la théorie des surfaces de translation et de leurs espaces de modules, ainsi que les dernières avancées dans ce domaine. Ce recueil regroupe quatre textes, tous issus des notes de cours qui ont été donnés pendant ces semaines, et illustre le lien fort entre théorie de Teichmüller et dynamique.

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ABSTRACTS

An introduction to K3 surfaces and their dynamics

Simion Filip 1

These notes provide an introduction to the geometry of K3 surfaces and the dynamics of their automorphisms. The notes are based on lectures delivered in Grenoble in July 2018, and in Beijing in July 2019.

Ruelle Resonances from Cohomological Equations

Giovanni Forni 47

These notes are based on lectures given by the author at the Summer School on *Teichmüller dynamics, mapping class groups and applications* in Grenoble, France, in June 2018 and at the Oberwolfach Seminar on *Anisotropic Spaces and their Applications to Hyperbolic and Parabolic Systems* in June 2019. We derive results about the so-called Ruelle resonances and the asymptotics of correlations for several classes of systems from known results on cohomological equations and invariant distributions for the respective unstable vector fields. In particular, we consider linear pseudo-Anosov diffeomorphisms on surfaces of higher genus, for horocycle flows on surfaces of constant negative curvature and for partially hyperbolic automorphisms of Heisenberg 3-dimensional nilmanfolds. Ruelles resonances for linear pseudo-Anosov maps with applications to the cohomological equation for their unstable translation flows was recently studied in depth by F. Faure, S. Gouëzel and E. Lanneau [9] by methods based on the analysis of the transfer operator of the pseudo-Anosov map. Ruelle resonances for geodesic flows on hyperbolic compact manifolds of any dimension and of partially hyperbolic automorphisms of Heisenberg 3-dimensional nilmanfolds are studied by general results of Dyatlov, Faure and Guillarmou [7] and Faure and Tsujii [10] based on methods of semi-classical analysis. These works do not derive results on cohomological equations for unstable flows or horospherical foliations of these systems.

Three lectures on square-tiled surfaces
Carlos Matheus 77

This text corresponds to a minicourse delivered on June 11, 12 & 13, 2018 during the summer school "Teichmüller dynamics, mapping class groups and applications" at Institut Fourier, Grenoble, France.

In this article, we cover the same topics from our minicourse, namely, origamis, Veech groups, affine homeomorphisms, and the Kontsevich-Zorich cocycle.

Mirzakhani's work on earthquake flow
Alex Wright 101

The Teichmüller unipotent flow can be defined concretely on certain moduli spaces of singular flat surfaces by shearing polygonal presentations of the surfaces. Thurston's earthquake flow on moduli spaces of hyperbolic surfaces is more mysterious. Both flows have deep and important connections to other areas of mathematics.

In this expository survey we give a geometric account of the main ideas behind Mirzakhani's theorem relating these two flows. Our presentation avoids some technical prerequisites that featured in the original more analytic presentation.