

The legacy of Jean-Pierre Kahane

Yves Meyer

Once Jean-Pierre Kahane entered the stately edifice of French mathematics, he immediately opened the windows. The harmonic analysis seminar that he established at Orsay (University of Paris XI) welcomed with a joyous eclecticism mathematicians from around the world and mathematics that was, at the time, often considered most unorthodox. It was at Orsay, thanks to J-P. Kahane, that I had the good fortune to meet R. Coifman, Ch. Fefferman, S. Hartman, H. Helson, Y. Katznelson, W. Rudin, E. Stein, G. Weiss, A. Zygmund, and so many other mathematicians from all corners of the globe. During the worst moments of the cold war, the Kahane seminar was neutral territory where lasting friendships were established between mathematicians from the East and the West.

J-P. Kahane's extensive interests and wide-ranging curiosity represents the spirit of the Age of Reason and the tradition of the Encyclopedia. He taught me that mathematics thrives only in contact with other sciences and the broader concerns of the times. J-P. Kahane has a keen nose for talent. For example, he recognized well before the rest of us the importance of Jean Ecalle's work.

My reference to the Age of Reason and Diderot's work applies also to J-P. Kahane's thought and style. The elegant concision and lucid exposition of the 18th century masters can be seen throughout J-P. Kahane's writing and lectures.

Friendship has played a central role in J-P. Kahane's scientific activities. Among his friends and collaborators, I must first mention Raphaël Salem. Together they wrote "Ensembles parfaits et séries trigonométrique," which was for years my bedtime reading. J-P. Kahane's thesis and his beautiful work on mean-periodic functions have been an inspiration for my own re-

search. I had the great pleasure to discover that the “numbers of Pisot and Salem” play a role in the behavior of certain mean-periodic functions. The friendship between J-P. Kahane and Y. Katznelson led to some wonderful discoveries. Here are two that I find particularly impressive. J-P. Kahane and Y. Katznelson formulated and proved the converse of the theorem of Paul Lévy: Any function that operates on the Wiener algebra is analytic. Laurent Schwartz was always amazed by this result. In 1955 Kahane went to Tunis and delivered a lecture on a preliminary result. He was able to find an absolutely convergent trigonometric series f such that the Fourier expansion of $|f|$ was not absolutely convergent. I was still a high-school student, but I was sitting there and I am still keeping today a vivid memory of that lecture. My destiny was written there. Another Kahane–Katznelson result that I am very fond of concerns the Fourier coefficients of continuous functions. They showed that “one can expect no more than that they are square summable.” In fact, they proved that each positive, square-summable series is less than or equal to the absolute values of the Fourier coefficients of a certain continuous function defined on the unit circle. I find the beauty and simplicity of this result fascinating.

The reference to Paul Lévy leads me to mention an important discovery about Brownian motion. J-P. Kahane showed that, for almost every Brownian trajectory, there exists a set of Hausdorff dimension one of “slow points.” These are the instants when the Brownian particle seems to “catch its breath” before setting off again on its random movements. The slow points are characterized by a Hölder exponent of $1/2$.

Worthy heir of the Enlightenment, J-P. Kahane has shown us that mathematics can and must serve both the advancement of knowledge and the improvement of life. Paradoxically, J-P. Kahane has succeeded in his project to open mathematics by working from the core of “pure mathematics.” By giving mathematics such a new direction, J-P. Kahane represents one of the most original spirits that I know.

Yves Meyer