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MATHEMATICS AND HYDRAULICS BETWEEN TURIN AND FERRARA IN THE 18TH CENTURY: THE WORKS BY F. D. MICHELOTTI AND T. BONATI

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Abstract. — During the 18th century mathematical studies devoted to hydraulic and hydrodynamics applications become relevant. The importance of studies both from a theoretical and a practical point of view is well documented by a remarkable increase of papers about the motion of waters. Many mathematicians were involved in this research field. They were asked to describe the motion of waters by means of mathematical formulas. Unfortunately, the motion of water can't be described by Euler's equations. That's why in this period many practical experiments were conducted in order to find a better description of the motion of waters in rivers and streams. After a brief overview of the condition of Italian studies about hydraulics and hydrodynamics in the 17th century, we will focus on case studies of Turin and Ferrara. In 1763 the King Carlo Emanuele III of Savoy financially supported the construction of a laboratory for hydraulic experiments in Turin and appointed Francesco Domenico Michelotti (1710–1787), professor of hydraulics at the university of Turin, with these experiments. In the same period another mathematician from Ferrara, Teodoro Bonati (1726–1820), was involved in similar studies and experiments on behalf of the Papal States. The scientific relationship

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Mots clefs. — Mathématiques appliquées au mouvement de l'eau, enseignement de l'hydraulique dans les universités et collèges italiens, consultant en hydraulique, Francesco Domenico Michelotti, Teodoro Bonati, expériences sur l'eau.

between Michelotti and Bonati will be presented in this article, starting from the examination of the correspondence between the two mathematicians in the period 1768–1772, soon after the publication of the first volume of the *Sperimenti idraulici* by Michelotti.

Résumé (Mathématiques et hydraulique entre Turin et Ferrare au XVIII^e siècle : les travaux de F. D. Michelotti et de T. Bonati)

Le XVIII^e siècle a été marqué par un intérêt croissant pour les études mathématiques concernant les applications hydrauliques et hydrodynamiques. L'importance de ces études, tant d'un point de vue théorique que pratique, est documentée par un grand nombre d'œuvres décrivant le mouvement des eaux par des formules mathématiques. Les équations formulées par Euler ne pouvant rendre compte de la complexité du mouvement des eaux dans les rivières et les canaux, la conduite d expériences pratiques s'est avérée nécessaire. Après une rapide vue d'ensemble des études italiennes concernant l'hydraulique et l'hydrodynamique au cours du XVII^e siècle, notre attention se focalisera sur les études menées à Turin et Ferrare. En 1763 le Roi Charles Emmanuel III de Savoie finance la construction d'un laboratoire pour des expériences hydrauliques à Turin tout en confiant sa direction à Francesco Domenico Michelotti (1710–1787), professeur d'hydraulique à l'université de Turin. Au même moment, un autre mathématicien de Ferrare, Teodoro Bonati (1726–1820) mène des études et des expériences au nom de l'État pontifical. Nous examinerons la relation scientifique entre Michelotti et Bonati aux moyen de la correspondance qu'entretiennent les deux hommes entre 1768 et 1772, à la suite de la publication du premier volume de *Sperimenti idraulici* de Michelotti.

1. INTRODUCTION

At the beginning of the 18th century the science of waters represented a new section of mathematical sciences, whose principles and topics came from physics and whose method of research was based on observations and experiments, according to the Galilean tradition from which it derived. In such a discipline two different parts—theoretical and practical—had to "coexist". Two different professional figures corresponded to these two different traditions: the technical consultant, usually a mathematician, who was asked to find solutions to problems of practical hydraulics, and the so-called "perito", an expert, who had to put it into practice. The role of the hydraulic consultant was often played by the most relevant members of the Galilean school. [1]

As far as studies on fluvial hydraulics, Italy has led the field since the 16th century, thanks particularly to the works by Galileo and his school. Since

[1] Many works published in the last twenty years deal with the science of waters in Italy: Fiocca [1998]; Fiocca et al. [2003]. On the theoretical contribution of Italian scientists to the science of waters see Maffioli [1994].

the twenties of the 17th century, Benedetto Castelli (1578–1643) began to study problems related to practical hydraulics. As a consultant for the Papal States he was involved in the debate regarding the regulation of the rivers between the provinces of Bologna and Ferrara. The result of these studies was the publication of the work, *Della misura delle acque correnti* (Rome, 1628). The treatise by Castelli deals with specific issues related to running waters, such as determining the mathematical relationship between the section of a river and its speed, flooding rivers and streams and methods for reducing or preventing floods. The text consists of two parts, the second provides an exact formulation of the law of continuity of fluid motion:

> For ideal fluids, in a continuous current in continuous motion the flow rate is constant in each section. [2]

In the Papal States the main centres for hydraulic studies were Bologna and Ferrara and the main representatives had local origins. Since the end of the 17th century the hydraulic tradition in Bologna was carried on first by Gian Domenico Cassini [3] (1625–1712) and Domenico Guglielmini (1655–1710), then by Eustachio (1674–1739) and Gabriele (1681–1761) Manfredi, Vittorio Francesco Stancari (1678–1709) and later by Giuseppe Venturoli (1768–1846). The origin of the hydraulic tradition in Ferrara may be traced back to Giambattista Aleotti (1546–1636) and Niccolò Cabeo (1586–1650). [4] During the 18th century the main representatives were Romualdo Bertaglia (1688–1763) and Teodoro Bonati (1726–1820). [5]

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2 Thanks to the Galilean school, Tuscany attracted many scientists from other parts of Italy, such as Castelli, Giovanni Alfonso Borelli (1608–1679), Famiano Michelini (1604–1665), Guido Grandi (1671–1742), Leonardo Ximenes (1716–1786), in addition to local scientists like Vincenzo Viviani (1622–1703), Tommaso Perelli (1704–1783), Vittorio Fossombroni (1754–1844), Vincenzo Brunacci (1768–1818). See Barsanti [1988]; Barsanti & Rombai [1987]; Maccagni [1987].

3 See Pallotti [1983]. Between the fifties and the sixties of the 17th century Gian Domenico Cassini was the leading actor of the Bolognese delegation. He took part in the conferences in Rome promoted by the Holy Congregation of Waters and wrote many hydraulic papers about the river Reno, who were collected in Collectives [1682]. On the figure of Gian Domenico Cassini see Giuntini [2001–2; 2006–2]. The role played by Eustachio Manfredi as hydraulic consultant for the city of Bologna clearly emerged by his correspondence with Guido Grandi in the period 1701–1738. See Basta [1992]; Forlani [1992].

4 Giambattista Aleotti worked in Ferrara as architect and hydraulic engineer during all his life. See Fiocca [1998]; Fiocca et al. [2003]. On the Jesuit Niccolò Cabeo see Fiocca [2002].

5 See Borgato et al. [1992]. Romualdo Bertaglia was a surveyor and hydraulic technician. Since the first decades of the 18th century he took part in many visits to the

As regards the motion of waters, a general theory that enabled a description of even easily observable phenomena did not exist. The lack of valid instruments to measure the velocity of waters contributed to support an empirical and local method. The technicians had to apply some empirical formulas, by adapting them to suit concrete situations. The greatest difficulty, using both equations and empirical formulas, was represented by the whirling motion of waters that flow in rivers and canals. Euler's equations for fluid dynamics, expounded in three essays published in the *Mémoires* of the Berlin Academy in 1757, could not be useful to describe the motion of waters because they referred to ideal conditions and did not consider whirling motions and frictional forces of the water in the river bed. Euler himself was aware of the limits of his work. In his essay about fluvial hydraulics, entitled *Recherches sur le mouvement des rivières*, he said:

> C'est peu de chose ce que les Auteurs ont écrit jusqu'ici sur le mouvement des rivières, et tout ce qu'ils en ont dit n'est fondé que sur des hypothèses arbitraires, et souvent même tout à fait fausses. [...] pour chercher le mouvement de l'eau dans une rivière, il faut abandonner les hypothèses auxquelles on a attaché jusqu'ici toutes les recherches hydrauliques, pour remonter aux premiers principes de Mécanique, par lesquels tous les mouvemens des corps tant solides que fluides sont déterminés. [6]

The motion of waters did not need the knowledge of refined analytical tools (partial differential equations, calculus of variation and so on), that were developed during the 18th century. In 1777 Joseph Louis Lagrange, writing to Anton Maria Lorgna, said that the principles of this discipline were still vague and that there still did not exist a geometric theory on this subject:

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waters on behalf of the city of Ferrara. See [Lugaresi 2014, pp. 213–216]. Also the Duchies of Modena and Mantua had a local tradition in hydraulic studies. Geminiano Montanari (1633–1687), Domenico Corradi d'Austria (1677–1756) and Giambattista Venturi (1746–1822) came from Modena. Geminiano Montanari was professor of mathematics at the university of Bologna from 1664 to 1679, then he moved to Padua where he kept the chair of astronomy and meteors. See Cattelani & Barbieri [1992]. See also [Maffioli 1994, pp. 129–163]. Domenico Corradi d'Austria was hydraulic engineer and mathematician of the Duke of Modena. He was involved in many problems related to the waters. See Pantanelli [1911]; [Pepe 1981, pp. 84–85]. On the figure of the Modenese Giambattista Venturi see Spaggiari [1984].

6 [Euler 1767, pp. 101–102]. In the middle of the 18th century fluid mechanics was organized as autonomous discipline thanks to the works by Daniel Bernoulli, *Hydrodynamica* (1738), Johann I Bernoulli, *Hydraulica* (1742), D'Alembert, *Traité de l'équilibre et du mouvement des fluides* (1744) and *Essai d'une nouvelle théorie de la résistance des fluides* (1752). The major contribution in this field came from three memoirs by Euler that contained the general equations of hydrodynamics for incompressible fluids. See Euler [1757a;b;c].

> With the exception of infrequently used general principles, I have found too much vague reasoning and experience, that cannot be used as a foundation for a strict and geometric theory. This science may be compared to practical medicine, which, despite its importance and the many things that have been discovered in anatomy, chemistry and natural history, has not progressed since the times of Hippocrates, in fact, it may even have regressed. [7]

Lagrange's words clearly shed light on the prevalence of practical point of view in the study of hydraulic matters, as will be explained in the next sections of this work.

2. MATHEMATICAL STUDIES APPLIED TO THE MOTION OF WATERS IN ITALY (17TH-18TH CENTURY)

In Italy subjects related to the regulation of waters had long-standing roots. Just think of the Roman systems of water supply, the building of irrigation and navigable canals or the reclamation of wide marshlands since the Middle Ages.

From a geographical point of view, the main locations of hydraulic interventions were in northern and central Italy. From a political point of view, the main states involved were the Grand Duchy of Tuscany, the Papal States, with particular focus on the provinces of Bologna, Ferrara and Ravenna, the Republic of Venice and to a lesser extent the Austrian Empire, whose territory extended to Lombardy. From a hydraulic point of view, the main interventions dealt with the regulation of some rivers and streams—the prime example was represented by the river Reno, whose path was very irregular in the Po river plane between the provinces of Bologna, Ferrara and Ravenna—, the reclamation of wide marshlands (such as the Pontine marshes in Lazio or some Tuscany wetlands like the

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7 "Fatta eccezione per qualche principio generale la cui applicazione ha raramente luogo, non vi ho trovato che ragionamenti ed esperienze troppo vaghi per poter servire da fondamento ad una teoria rigorosa e geometrica. Fin'ora è di questa scienza come della medicina pratica che, nonostante la sua estrema importanza e nonostante le belle scoperte che sono state fatte in Anatomia, in Chimica, in Storia Naturale ecc. non è più progredita dal tempo di Ippocrate, se anche non è regredita". J. L. Lagrange, *Œuvres*, Paris, Gauthier-Villars, t. XIV, p. 260. A fundamental contribution to the study of the history of fluids mechanics was given by Truesdell in Truesdell [1954]. Some years later, Roger Hahn provided a wide summary of the development of hydrodynamics in the 18th century in Hahn [1965]. General studies on hydrodynamics have been resumed by different authors over the past few years: Blay [2007]; Calero [2008]; Darrigol [2005]; Darrigol & Frisch [2008]. Hydrodynamics in France has been recently studied by A. Guilbaud. See Guilbaud [2007; 2008; 2012; 2013].

lake of Bientina or the Val di Chiana), the sediment of debris at the mouth of harbours or in the lagoon of Venice. [8]

Hydraulic problems were often not merely a question of solving mathematical problems, but a political issue. Hydraulic disputes among Italian states have been countless. Since the 16th century Italian states began to appoint their own hydraulic consultants in order to study the specific hydraulic problem of their territory and to represent their interests during the long debates that took place soon after the so-called "visite alle acque" (visits to the waters). Many water-related affairs had historic significance. The most important one, represented by the regulation of the river Reno in the Po valley between the provinces of Bologna and Ferrara, could be considered the foundation of Italian hydraulic science. [9] The Papal States, by means of its delegates (usually a Cardinal), chaired those meetings. The Papal delegates, helped by some neutral specialists, organized periodical surveys, to which the delegates of all the States involved in the dispute took part. As regards the problem of the regulation of the river Reno, many different Italian states were involved in the debate on the proposal for the placing of the river Reno into the river Po. The visits to the waters were often named after the Cardinal who chaired them. For instance during the so-called "Visita Rinuccini" (Monsignor Giovanni Rinuccini) there were ten mathematicians, who represented different Italian states: the Papal mathematicians Guido Grandi and Celestino Galiani, Eustachio and Gabriele Manfredi together with Francesco Maria Zanotti (1692–1777) for

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8 In Maffioli [1994] the author traces the development of the science of waters in Italy in the period 1628–1718, by focusing on two prime examples: the problem of the floods of the river Reno between the provinces of Bologna and Ferrara and the problem of the Venetian lagoon. An historical reconstruction of the events related to the river Reno is given by Bertoldi [1807]; see also Collectives [1983a;b]; Giacomelli [1983]. As regards the Venetian lagoon, in 1720 Bernardino Zendrini began a general report of the Venetian hydrological system. The work was completed in 1726, but it was published only after his death Zendrini [1811]. During the 19th century Pietro Paleocapa gave a report of the lagoon of Venice Paleocapa [1867]. An overview of the hydraulic problems in Tuscany can be found in Barsanti & Rombai [1986].

9 The Reno is an Italian river and before its deviation it was the main right tributary of the river Po. This river had a very irregular path in the Po river plane between the provinces of Bologna, Ferrara and Ravenna. In its lower course the Reno receives the water of numerous streams, some of which are seasonal, that carries great quantities of stones and gravel. That's why its floods were so frequent and dangerous. From the 16th century on this problem became unacceptable and the authorities whose job was to deal with this damage decided to tackle the situation in a definitive way. However, this was just the beginning of a long series of quarrels that involved the provinces of Bologna, Ferrara and Ravenna. The problem of the regulation of the river Reno and a wide description of the visits to the waters during the 16th, the 17th and the 18th centuries have been deeply studied in Lugaresi [2014].

the Bolognese delegation, Domenico Corradi D'Austria and Bernardino Zendrini for the city of Modena, Giovanni Ceva and Doriciglio Moscatelli Battaglia for the city of Mantua, Jacopo Marinoni (1676–1755) for the Austrian Empire. [10]

Italian scientists became advisors of the rulers in many important issues related to hydraulic problems (reclamations, planning and constructions of hydraulic devices ...) and the figure of the superintendent of the waters obtained also an administrative acknowledgement, as we can see in Tuscany, in the Papal States and in the Republic of Venice. From a professional point of view, the hydraulic consultants had different trainings and came from different backgrounds. Some of them had studied medicine, like D. Guglielmini, B. Zendrini and T. Bonati, some others undertook juridical studies before devoting themselves to mathematics, such as Geminiano Montanari and Eustachio Manfredi. Some other scientists came from the military career and were appointed hydraulic consultants, like D. Corradi d'Austria in Modena and Anton Maria Lorgna in Verona. Another relevant group of scientists belonged to religious orders and played an important role in the development of hydraulic science by means of their lectures and treatises. Among them we can mention the Piarist Famiano Michelini, the Camaldolese Guido Grandi in Pisa, the Jesuits N. Cabeo in Ferrara, Ruggiero Giuseppe Boscovich (1711–1787) in Rome and Pavia, Giovanni Antonio Lecchi (1702–1776) in Milan, Leonardo Ximenes in Siena, the Barnabite Paolo Frisi (1728–1784) in Pisa and Milan.

A useful historical source for the main hydraulic problems discussed by Italian mathematicians between the 17th and the first decades of the 18th century is the first Florentine edition of the collection on the motion of waters (*Raccolta d'autori che trattano del moto dell'acque*). [11] This *Raccolta* contains both theoretical and practical contributions to the science of waters,

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10 The report of the visit was the paper entitled *Visita fatta negli anni 1719, e 1720 da Monsignor Rinuccini Commissario Apostolico e dal Sig. Generale di Latterman Commissario per Sua Maestà Cesarea, e Cattolica coll'intervento de Commessarj d'altri Principi interessati, e loro respettivi Matematici per la pretesa introduzione del Reno in Po.* A copy of the manuscript is kept in the Historical Archive of the city of Ferrara. See [Lugaresi 2014, p. 181]. The hydraulic technicians of the Duchy of Mantua were Giovanni Ceva (1647–1734) and Doriciglio Moscatelli Battaglia (1663–1739). Mathematical contributions by Giovanni Ceva have been studied in Mercanti [2004]. See also [Pepe 1981, pp. 85–86]. The figure of the hydraulic engineer Doriciglio Moscatelli Battaglia has been examined in Ceriali & Moraschi [1992].

11 Collectives [1723]. The collections on the motion of waters have been studied in Lugaresi [2015]. As regards the first edition of the collection see [Lugaresi 2015, pp. 203–215]. See also Lugaresi [2017b].

published in three volumes in a chronological order.[12] The collection of essays on the motion of waters belongs to a branch of research that directly derived from the Galilean tradition. The first volume contains works by Archimedes,[13] Lorenzo degli Albizi,[14] Galileo Galilei,[15] his former students Benedetto Castelli,[16] Vincenzo Viviani,[17] and others who were directly inspired by their studies and by their approach to doing science, including Famiano Michelini,[18] Giovanni Alfonso Borelli,[19] Geminiano Montanari[20] and Gian Domenico Cassini.[21] The second volume contains other hydraulic works by D. Guglielmini (*Misura delle acque correnti*; *Della natura dei fiumi*; *Scritture intorno all'affare del Reno*)[22] and G. Grandi

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12 The three volumes were prepared by Tommaso Buonaventuri (1675–1731), who served as director and editor of the printing works of the Grand Duchy of Tuscany from 1713 to 1723. Among his collaborators on this project were the scholars Giovanni Gaetano Bottari (1689–1775) and Benedetto Bresciani (1658–1740), and the mathematician G. Grandi. From 1714 Guido Grandi was superintendent of the waters of the Grand Duchy of Tuscany. The correspondence between Grandi and G. Manfredi has been published in Franci [1984]; Giuntini [1993–1], while that between Grandi and Celestino Galiani (1681–1753) was published in Palladino & Simonutti [1989]. Grandi was also in relationship with Hermann. This correspondence has been published in Mazzone & Roero [1992]. On the correspondence between G. Manfredi and Giovanni Poleni, see Giuntini [1990–1].

13 [Collectives 1723, vol. I, pp. 1–22]. Archimedes, *Delle cose che stanno nell'acqua*, translated in Italian by G. G. Bottari, with notes by G. Grandi.

14 [Collectives 1723, vol. I, pp. 23–34]. Lorenzo degli Albizi, *Dialogo sopra i paduli di Pisa.*

15 [Collectives 1723, vol. I, pp. 35–108]. Galileo Galilei, *Discorso intorno alle cose che stanno in su l'acqua, o che in quella si muovono*; *Lettera sopra il fiume Bisenzio a Raffaello Staccoli.*

16 [Collectives 1723, vol. I, pp. 109–212]. Benedetto Castelli, *Della misura dell'acque correnti.*

17 [Collectives 1723, vol. I, pp. 347–390]. Vincenzo Viviani, *Discorso intorno al difendersi da' riempimenti, e dalle corrosioni de' fiumi applicate ad Arno in vicinanza della città di Firenze*; *Relazione intorno al riparare, per quanto possibil sia, la città, e campagne in vicinanza della città di Firenze.*

18 [Collectives 1723, vol. I, pp. 213–272]. Famiano Michelini, *Trattato della direzione de' fiumi.*

19 [Collectives 1723, vol. I, pp. 273–318]. Giovanni Alfonso Borelli, *Discorso sopra la laguna di Venezia*; *Relazione sopra lo stagno di Pisa.*

20 [Collectives 1723, vol. I, pp. 319–346]. Geminiano Montanari, *Il Mare Adriatico e sua corrente e la naturalezza de' fiumi scoperta, e con nuove forme di ripari corretta.* Since 1678 Geminiano Montanari was in Padua as a professor at the local university, but also as a consultant of the Magistrate to the waters of the Republic of Venice. He had a practical approach to hydraulic problems. See [Maffioli 1994, pp. 152–163].

21 [Collectives 1723, vol. I, pp. 391–406]. Gian Domenico Cassini, *Scritture concernenti il regolamento dell'acque del Bolognese e del Ferrarese.*

22 [Collectives 1723, vol. II, pp. 1–433].

(*Trattato geometrico del movimento dell'acque*; *Riflessioni su alcune controversie relative al fiume Era*).[23] The third volume was almost entirely devoted to hydraulic papers about the river Reno. It contains many writings by Eustachio Manfredi (*Scritture del Signor Eustachio Manfredi*),[24] among which we can quote *Memoria che contiene le ragioni per l'unione dell'acque del Reno di Bologna col fiume Po*.[25] In the same volume there are two more treatises by Jean Picard (1620–1682),[26] translated from French, and by Tommaso Narducci (1679–1766), respectively.[27]

As we can see just from reading the titles of the writings, the main themes discussed deal with three instances: the diversion of the Reno from the Po in the Ferrara region, the diversion of rivers from the lagoon of Venice and the reclamation of some Tuscan wetlands.[28]

23 [Collectives 1723, vol. II, pp. 435–713]. The treatise by Guido Grandi, *Trattato geometrico del movimento dell'acque*, is the only unpublished work and it was written in 1723 especially to be inserted in the collection, as we can read in some letters Tommaso Buonaventuri wrote to Guido Grandi in the period soon after the publication of the *Raccolta*. All the other writings of the *Raccolta* have already been printed before. See [Lugaresi 2017b, pp. 205–208; 212–216].

24 [Collectives 1723, vol. III, pp. 3–418].

25 [Collectives 1723, vol. III, pp. 329–418]. E. Manfredi originally wrote this paper in French, *Mémoire qui contient les raisons pour la junction du Reno avec la rivière du Po* (Paris, 1719) in order to have the support of the Académie des Sciences in the suit of the Bolognese waters. The paper by Manfredi was translated in Italian and annotated by Guido Grandi. Since 1692 E. Manfredi kept the lecture of mathematics in the Studium of Bologna. In 1704 he became also Superintendent to the waters of Bologna, succeeding to Guglielmini who moved to Padua. Some papers by E. Manfredi were written to confute the arguments proposed by the adversaries of the Bolognese project. Among Manfredi's enemies, we can quote the commissioner of the artillery of the Duke of Modena, the mathematician Domenico Corradi d'Austria, who wrote a paper entitled *Effetti dannosi che produrrà il Reno se sia messo in Po di Lombardia* (Modena, 1717). See Pantanelli [1911]. Another opponent of Manfredi's project was Giovanni Ceva, commissioner of the Duke of Mantua, who published *Conseguenze del Reno, se coll'aderire al progetto de' Signori Bolognesi, si permettesse in Po grande* (Mantua, 1716). See Mercanti [2004]. The debate between Manfredi and Corradi frequently recurs in the letters E. Manfredi wrote to Grandi. See Basta [1992]; Forlani [1992].

26 [Collectives 1723, vol. III, pp. 489–526]. Jean Picard, *Trattato del livellare*. Picard's treatise *Traité du nivellement* was published in Paris in 1684 by Philippe de la Hire soon after the death of Picard. This was the only paper by a foreign author to be put in the *Raccolta*.

27 [Collectives 1723, vol. III, pp. 527–556]. Tommaso Narducci, *Il paragone de' canali: considerazione necessaria per ben regolare gli scoli delle campagne*. Narducci was one of the most important pupil of Guido Grandi. His hydraulic treatise was very appreciated also by Charles Bossut: [Bossut 1795–1796, vol. II, p. 457]. See [Lugaresi 2014, pp. 13–14; 25–26]. See also [Lugaresi 2017b, pp. 217–220].

28 See Lugaresi [2015].

The issues related to the waters were always very topical in the Republic of Venice, such as diversion of rivers from the lagoon, excavation of lagoon canals, supervision of the main rivers (Adige, Piave, Brenta...), creation of inland waterways. [29] The Venetians had an experienced technical staff at their disposal, the Magistrate to the waters ("Magistrato alle acque"). In the Republic of Venice the office of superintendent to the waters was separate from university education. Geminiano Montanari was one of the first mathematicians of the university of Padua who collaborated with the Magistrate to the waters of Venice in order to study hydraulic problems related to the rivers that flowed into the lagoon. After Montanari's death, the task of hydraulic consultant for the Republic of Venice was held by D. Guglielmini, who accepted the proposal by the Riformatori of the Studium and moved from Bologna to Padua in 1698. When he was called to Padua, Guglielmini was already a famous mathematician. [30]

During the last decade of the 17th century Guglielmini published two important books on general and river hydraulics: *Aquarum fluentium mensura* (Bologna, ex typographia Pisariana, 1690) and *Della natura dei fiumi* (Bologna, per gli eredi di Antonio Pisarri, 1697). In the *Aquarum fluentium mensura* Guglielmini tried to order and organise the subject of motion of waters according to the geometric method. [31] During his last Bolognese years, Guglielmini's reputation grew and reached its peak in 1697 with the publication of *Della natura dei fiumi*. Guglielmini studied the efflux of a fluid from a vessel and described water as an aggregate of smooth and small spherical particles. Practical experiences should be strictly connected with theoretical improvement: mathematical studies supported the birth of hydrodynamics, i.e., rational mechanics applied to fluids. Fluid dynamics could solve practical problems related to the motion of ideal fluids. In a few cases, the behaviour of water was similar to that of an ideal fluid, but a great number of problems related to water couldn't be solved because of lack

29 In 1641 also Benedetto Castelli had been asked for an advice about the merge of the rivers in the lagoon of Venice: Castelli's report was published in *Considerazioni intorno alla laguna di Venezia* in [Castelli 1660, pp. 99–142].

30 The Venetian hydraulic tradition continued along the 18th century with Anton Maria Lorgna (1735–1796) and Simone Stratico (1733–1824). As regards the development of studies devoted to hydraulic themes in other Italian northern states, in Lombardy during the 18th century the main figures were the mathematicians Giovanni Antonio Lecchi and Paolo Frisi and the engineer Antonio Tadini (1754–1830). See Barbarisi [1990]; Bigatti [1995]. In Piedmont hydraulic science had a slower development that dated back to the mid of the 18th century, as we will see in the next paragraph, thanks to the studies by G. B. Beccaria and F. D. Michelotti and the creation of the laboratory of the Parella.

31 See [Maffioli 1994, pp. 195–207].

of proper mathematical instruments. [32] In practical hydraulics, the use of physical models was very important, as well as the historical results obtained during the centuries. In the first half of the 18th century the contributions to the development of hydraulics thanks to Venetian scientists, like Giovanni Poleni (1683–1761), Bernardino Zendrini (1679–1747) and Jacopo Riccati (1676–1754) became quite relevant.

After the death of Guglielmini, two important scientists emerged in the Veneto region in the field of hydraulics: B. Zendrini and G. Poleni. Their studies were deeply influenced by the presence of Jacob Hermann (1678–1733) at the university of Padua. [33] Zendrini performed some important experiments on the velocity of the currents. His research was focused on trying to find the equation of the curve of velocities taking resistances into account, following a mathematical—empirical approach. From 1720 to 1747 the Venetian authorities appointed Zendrini to superintend all the hydraulic matters of the Republic. As hydraulic consultant, Zendrini devoted many studies to the Venice lagoon. His main contribution to the study of the motion of waters was the treatise *Leggi e fenomeni regolazioni ed usi delle acque correnti* Zendrini [1741]. [34]

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32 During the 17th and the 18th centuries many Italian scholars of hydraulics suggested and tested a selection of new instruments to measure flow velocity at different depths. These devices were based on different methods: the fixed hydraulic speedometers and the floating hydraulic speedometers. As regards the fixed hydraulic speedometers, one of the most long-lived ones was the so-called "pendulum ball" with quadrant ("palla a pendolo" con quadrante) by Guglielmini. The Piedmontese Michelotti tested a new instrument, called "Michelotti's paddle wheel" ("ruota con palmette di Michelotti"). This device, counting the number of turns made by the wheel in a given time, gave the velocity of the flow in the point where it hit the blade of the wheel. See Michelotti [1767–1771]. Some years later the Jesuit Leonardo Ximenes invented a new hydraulic speedometer, that he called "hydraulic fan" ("ventola idraulica"), see Ximenes [1780]. As regards the floating hydraulic speedometers, one of the oldest ones was the hydrometric rod. The Milanese G. A. Lecchi invented a "floating cylinder" ("cilindro natante"), see [Lecchi 1765, pp. 424–426]. A similar instrument, called "asta ritrometrica", i.e., a rod for measuring flow, was invented and used by T. Bonati, see Bonati [1784].

33 In 1707 the Leibnizian mathematician Jacob Hermann was appointed to the Chair of Mathematics in the university of Padua, where he remained until 1713. He was one of the favourite pupil of Jacob Bernoulli. Thanks to Leibniz, from 1713 to 1725 Hermann taught mathematics at Frankfurt on the Oder, then he moved to St. Petersburg where he was called by Peter the Great in order to found a Scientific Academy. The figure of Jacob Hermann and his relationship with the Venetian scientific environment has been studied in Mazzone & Roero [1997]. See also Fellmann & Mikhailov [2016]; Mikhailov [1996]. The chair of mathematics in Padua, after Hermann's departure, was held by Nicolaus I Bernoulli in the period 1716–1719 and then by Poleni, from 1720 to 1761.

34 Bernardino Zendrini (1679–1747) was pupil of Domenico Guglielmini in the faculty of medicine at the university of Padua. Thanks to Hermann, Zendrini approached

As is well known the new methods of the infinitesimal calculus began to spread in Italy in the last years of the 17th century thanks to Leibniz and his scientific heirs. The Leibnizian calculus began to be taught in the Italian Universities with public and private lessons. [35]

Another source for the diffusion of the Leibnizian calculus in Italy was represented by the articles published in the *Giornale de' letterati d'Italia* that contained many references to the applications of differential and integral calculus for the resolution of different problems of applied mathematics. [36]

In 1716 Jacob Hermann published *Phoronomia sive De viribus et motibus corporum solidorum et fluidorum libri duo*, which he wrote during his period as professor at the university of Padua. [37] The *Phoronomia* influenced Giovanni Poleni, an Italian scholar of mathematics and hydraulics who improved the studies of this subject. Thanks to the relationship with Hermann, Giovanni Poleni directed his studies towards a physico-mathematical field. Poleni studied the motion of waters with a physical approach. As an hydraulic experimenter, he arranged suitable gears with which he could verify his theoretical suppositions. [38]

In 1717 he published a short treatise about the motion of waters, *De motu aquae mixto.* The book is devoted to explain the motion of water by the theory

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the new analytical methods and applied them also to physical problems. See [Mazzone & Roero 1997, pp. 114–124]. See also Collectives [2007].

35 We can quote Hermann's private teaching in Padua or Grandi's one in Pisa. In 1708 the chair *Ad Algebram sive Analysin tam commune quam infinitorum* was created in Bologna and it was initially held by V. F. Stancari. On June 15 1708, Stancari defended twelve *Theses Physico-Mathematicas* about different topics, among which the doctrine of fluids. See [Mazzone & Roero 1997, pp. 443–445]. In 1709, after Stancari's death, the chair of infinitesimal analysis was held by G. Manfredi in private lessons. From 1720 to 1761 Manfredi taught infinitesimal analysis also in his public university course.

36 Some articles published in the *Giornale de' Letterati d'Italia* concerned hydraulics: Zendrini B., *Modo di ritrovare ne' fiumi la linea della corrosione, i quali si escavano negli argini composti di parti ammovibili, qualora il corso delle loro acque sia in qualche maniera alla direzione delle sponde obbliquo*, in *Giornale de' Letterati d'Italia*, t. 21 (1715), pp. 105–136; Verzaglia G., *Considerazioni sopra l'art. XVI del tomo V del giornale de' Letterati nel quale si tratta del problema inverso generale delle forze centrali nel voto, e di questa in un mezzo fluido, e resistente, presupposta qualsiasi legge delle resistenze*, in *Giornale de' Letterati d'Italia*; t. 6 (1711), pp. 411–440. The contribution of the mathematical community to the *Giornale de' Letterati d'Italia* has been studied in Roero [2012].

37 Hermann [1716]. The *Phoronomia* is devoted to the dynamics of solid and fluid bodies and deals with many problems discussed by Newton in the first two books of the *Principia.* In his treatise, Hermann uses geometrical methods, since they are more suitable for beginners. However, his knowledge of calculus is evident in the way in which he deals with infinitesimals. Guicciardini [1996]. See [Mazzone & Roero 1997, pp. 80–82; 183–188].

38 On the figure of Giovanni Poleni see Soppelsa [1988; 1997]; [Mazzone & Roero 1997, pp. 142–159].

of mixed motion ("motus mixtus"). Through the example of the lagoon of Venice Poleni described the ebb and flow of waters between the lagoon and the sea. [39] Poleni promoted the study of the motion of water by using real cases. His second treatise on this topic, *De castellis per quae derivantur fluviorum aquae habentibus latera convergentia liber*, was inspired by Poleni's study of the river Adige. [40] In 1716 Poleni began to work as a consultant in the subject of waters, under the command of a local office, the "Presidenza dell'Adige" of Padua. He was charged to superintend to the restoration of the banks and the rebuilding of an important overflow structure of the river. In his *De castellis* Poleni studied the hydraulic phenomenon of the derivation of the water of the river through a diversion canal. [41]

Poleni's hydraulic works were very appreciated by Jacopo Riccati, another leading figure in the Veneto scientific environment of the first decades of the 18th century. The figure of J. Riccati was very representative of the improvement of the new methods of the infinitesimal calculus, also in its applications to physical problems. Riccati's approach to the science of waters was more mathematical than physical. In the mid of 1710s Riccati helped Zendrini with his article on river resistances, that was published in 1715. [42] Between 1715 and 1727 Jacopo Riccati, supporter of the Newtonian point of view, was involved in a heated debate on the evaluation of the total momentum of a fluid in a vessel during discharge, in opposition to the Leibnizians Johann Bernoulli (1667–1748) and his son Daniel (1700–1782) and Pier Antonio Michelotti (1673?-1740). [43] The

39 Poleni's approach to the study of the lagoon was mathematical and experimental. He introduced the concepts of *aqua mortua* (i.e., the lagoon water), *aqua viva* (i.e., the sea water) and *motus mixtus* (i.e., when the lagoon water is in contact with the sea water) [Mazzone & Roero 1997, pp. 147–148]. See also [Maffioli 1994, pp. 351–363]; [Maffioli 2015, pp. 89–91].

40 In his works, Poleni described his experiments with outflow of water through different orifices by using analytical methods. In each experiment he measured the volume of rate of flow and the diameter of the contracted section of the jet.

41 Poleni planned a flood relief canal in the Paduan side of the river Adige in order to divert the amount of water during the floods. The sides of this fluvial structure had to resist the impetus of the river and their mouths had to keep a fixed shape and size in order to derive a defined amount of water. See [Maffioli 1994, pp. 371–375]. See also [Maffioli 2015, p. 91].

42 The mathematician of Castelfranco, Jacopo Riccati, was one of the greatest Italian experts in infinitesimal analysis. He was largely self-taught as a mathematician. Through the mediation of Zendrini, Riccati had a scientific relationship with Hermann. See [Mazzone & Roero 1997, pp. 125–142]. On Jacopo Riccati see Piaia & Soppelsa [1992]; Soppelsa [1997].

43 Pier Antonio Michelotti was spokesperson of Johann Bernoulli, the main representative of the continental school of the adherents of Leibniz, in the dispute with

dispute with Jacopo Riccati would be useful for Daniel Bernoulli, in the formulation of his famous hydrodynamic theorem. [44]

3. THE MOTION OF WATERS IN GOUVERNMENT INSTITUTIONS AND IN UNIVERSITIES

The interest in problems related to the waters involved specific organizations in different Italian states. In the Grand Duchy of Tuscany the institution appointed for the control and superintendence of public works was the "Magistratura dei capitani di parte guelfa". This office, reformed in 1549, was charged for the maintenance of streets and rivers. It had both an administrative aim and an environmental protection. It was composed by a college of ten members, two of which, chosen by the Grand Duke, remained in office with permanent contract and were called "Officials of the rivers" and "Officials of the ditches" (*Ufficiali dei Fiumi* and *Ufficiali dei Fossi*). [45] Many architects and engineers collaborated with the Magistratura, especially on technical problems about the science of waters. Another important scientific figure inside the Grand Duchy of Tuscany was represented by the "royal mathematician" (*matematico regio*), a role entrusted to the most eminent mathematicians, who, starting from Galileo, worked in Tuscany. [46]

The "Holy Congregation of Waters" (*Sacra Congregatio Aquarum*) was created by the Pope Sistus V on January 22nd 1587 with the seal *Immensa Aeterni Dei* in order to superintend to the works of reclamation and maintenance of the river banks of the Papal States. [47] The Congregation had

Jacopo Riccati. The problem of the outflow of water through orifices was studied also by Daniel Bernoulli in his *Exercitationes mathematicae* (1724). At that time, D. Bernoulli was still a consistent defender of his father. At the end of 1730s Daniel entered into a polemic with Johann Bernoulli and recognised the validity of Riccati's opinion, as he wrote in his *Hydrodynamica.* [Mikhailov 1996, pp. 233–246]. See also [Mazzone & Roero 1997, pp. 176–183]. The theme was also discussed by Daniel Bernoulli in his correspondence with Euler. See Fellmann & Mikhailov [2016].

44 Bernoulli [1738]. On the dispute between Jacopo Riccati and Daniel Bernoulli in the years 1723–24, we can find a thorough description in Mikhailov [1996]; Ventrice [1992]; [Mazzone & Roero 1997, pp. 80–82, 140–159, 176–188, 315–316].

45 The *Magistratura dei Capitani di parte guelfa* was suppressed in 1769. Its job was entrusted to a new magistrature called *Camera delle Comunità.*

46 After Galileo, the role of "royal mathematician" was played by Evangelista Torricelli, Vincenzo Viviani, Guido Grandi, Tommaso Perelli, Leonardo Ximenes, Pietro Ferroni.

47 The *Sacra Congregatio Aquarum* was suppressed in 1870 soon after the annexation of the Papal States to the Kingdom of Italy.

jurisdiction, in addition to waters and rivers, also over harbours, mills, ducts, gutters, canals. The Congregation attended the inspections made by the apostolic Visitor who was charged with knowing and trying to solve the disputes among different cities of the Papal States or among the Papal States and other Italian states. In 1518 the city of Bologna founded a hydraulic board, the *Assunti to the waters*, with the aim of providing for the maintenance of rivers and hydraulic devices of the Bolognese plain.[48] In the city of Ferrara hydraulic legislation still existed in the Middle Ages and continued in the next centuries until the enactment of the *Ordini et provigioni sopra i Lavorieri del Po e Ufficiali a quelli deputati* (1580) by means of the Duke of Ferrara, Alfonso II d'Este. These measures dealt with restoration of river banks, excavation and maintenance of public drainage canals. Before the devolution to the Papal States, in the Duchy of Ferrara the management of the waters was entrusted to a "ducal engineer".[49] After the devolution (1598), the Pope Clemente VIII confirmed the statutory rules.[50]

In the controversy about the river Reno the problems of territorial hydraulics were mainly debated by local magistratures and offices whose interests were consistently opposite. The city of Bologna was opposed to Ferrara and Ravenna in finding the better solution for the path of the Reno. Many private interests entered the debate, that's why not only mathematicians, but also notaries, judges of river banks, lawyers and administrators wrote hydraulic papers in order to support their ideas and to rebut the theses of opponents.[51]

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48 The Bolognese *Assunteria di Confini, acque e fiumi* had a technical role in the regulation of waters. In the 1680s the Assunti, aware of the need to have a mathematician at their disposal both to superintend the technicians' work and to teach the subject to people who wanted to enter the field, proposed to the Senate of Bologna to elect Domenico Guglielmini as Superintendent to the waters. The proposal was accepted in 1686. See [Maffioli 1994, p. 172].

49 The office was held by many important scientists, like Silvio Belli (1510?-1580), Francesco Patrizi (1529–1597) and Giambattista Aleotti. The later continued to collaborate with the city of Ferrara also after the devolution of the Duchy of Ferrara to the Papal States in 1598. See Fiocca [1998].

50 In 1752 it was established the "Congregazione dei Lavorieri". Under its jurisdiction there were the "judges of river banks" (*Giudici d'argine*), the technical-administrative structure devoted to the safeguard and surveillance of rivers, draining canals and hydraulic products. See [Cazzola 1998, pp. 23–27].

51 On these subjects many papers were written and published between the 17th and the 18th century. We can quote, for example, two polemic papers written by a Bolognese lawyer: Ascevolini G., *Alla sacra Congregazione delle Acque per la città di Bologna. Contro la città di Ferrara. Memoriale di risposta di fatto e di ragione*, Typis de Comitibus, 1717; Ascevolini G. & Manfredi E., *Alla sacra Congregazione delle acque per lo Reggimento*

As regards the management of the waters, a different approach was held by the Republic of Venice, where technical and administrative management of the lagoon waters was entrusted with a single office, i.e., the Magistracy to the waters. This Magistracy, created in 1501, had to superintend the works of reclamation, excavating, maintenance and regimentation of the Venetian lagoon and of the set of the rivers that flowed into it. The general monitoring of lagoon and rivers was under the jurisdiction of the Magistracy, but sometimes special superintendents were appointed in order to look after specific hydraulic questions. In 1677 the Senate of Venice approved the institution of the Magistracy of the Provveditori to the river Adige.[52] In 1720 the Senate of the Republic of Venice appointed Bernardino Zendrini mathematician and superintendent of the waters of rivers, lagoons and harbours of the Venetian state (*Sopraintendente alle acque dei fiumi, delle lagune e dei porti dello Stato Veneto*).

The interest in hydraulic studies is also confirmed by the introduction of the science of waters inside the universities:

> [Hydrometry concerns ...] the mensuration of water, and other fluid bodies, their gravity, force, velocity, quantity, &c. Hydrometry includes both hydrostatics and hydraulics. In the year 1694 a new chair, or professorship of hydrometry was founded in the university of Bologna in favour of Domenico Guglielmini, who had carried the doctrine of running waters, with respect to rivers, canals, dykes, bridges &c. to an unusual length.[53]

From 1680 to 1698 Guglielmini was lecturer in mathematics (*Ad mathematicam*) in the Studium of Bologna. In 1694 the title of this lecture changed in hydrometric mathematics (*Ad mathematicam hydrometricam*), since the university of Bologna decided to create a chair of hydrometry. From this moment on, hydrometry became an autonomous discipline,

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di Bologna. Contro la città di Ferrara. Risposta al progetto accennato nel memoriale di replica &c., Typis de Comitibus, 1718; and the replays by the lawyer of Ferrara: Pandolfi F. & Melella A., *Alla Sacra Congregazione delle Acque per la città di Ferrara. Contro la città di Bologna. Memoriale di fatto, e di ragione, con nuovo sommario*, Typis de Comitibus, 1718; Pandolfi F. & Melella A., *Alla sacra congregazione delle acque per la città di Ferrara. Contro la città di Bologna. Memoriale di replica di fatto, e di ragione da vedersi per special grazia*, Typis de Comitibus, 1718. Some of these polemic papers have been examined in [Lugaresi 2014, pp. 157–180; 191–203; 211–233].

52 Inside these offices there were members of the Venetian patriciate and some technical experts, called *proti*. Giovanni Poleni collaborated with the office of the Provveditori to the Adige in the years 1716–19.

53 *The Cyclopaedia or Universal Dictionary of Arts, Sciences and Literatures*, by Abraham Rees, vol. XVIII, London, 1819.

clearly separate from mathematics. Guglielmini was the first mathematician to keep this chair from 1694 to 1698. In 1698, when he left Bologna, the chair was held by Geminiano Rondelli (1652–1739).[54]

In the preface to the second edition of the treatise *Della natura dei fiumi* (1739) by D. Guglielmini, Eustachio Manfredi made some remarks as regards the job of the "perito" of waters and his cultural background:

> I refrained from entering so deep researches that it was essential to use the most sublime geometry to get to the bottom of it. In the profession of this art [the science of waters] the circumstances where higher geometry is required are very uncommon and those where algebraic calculus is necessary are even less [...] The science of waters requires undoubtedly every rule of carefulness. It should be practised by people who could have observed and learnt what were the most frequent facilities and difficulties [...] in theoretical books very little or nothing at all can be found as regards materials or manufactures of hydraulic works [...] It would be advisable that some expert engineer should write a methodical treatise that carefully teaches this discipline [...] I agree that in such matters blind practice has no use and abstract theory is useless. Perfection should consist of a judicious combination of them. We can be glad that in our times the wisest Italian hydraulic surveyors and engineers, aware of the need of combining their studies, had begun to appreciate the theoretical basis thanks to the reading of this book [Guglielmini's treatise].[55]

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54 The new subject was included in the rolls of the university of Bologna from 1695 to 1794. Many professors followed one another in the chair of hydrometry in Bologna during the 18th century: Eraclito Manfredi (1683–1759), the youngest brother of Eustachio and Gabriele, Eustachio Zanotti (1709–1782), nephew of Francesco Maria Zanotti, and Giovanni Antonio Pedevilla (1736–1808). The subject of waters has already been taught in universities and colleges as a section of the mathematical sciences. See Maffioli [1987]; [Maffioli 1994, pp. 248–249].

55 "Mi sono astenuto da entrare in ricerche talmente profonde, che per venirne a capo fosse indispensabile l'uso della più sublime geometria, e tanto più, che nella professione di quest'arte ho osservato non essere, che assai rari i casi, ne' quali faccia d'uopo ricorrervi, e molto meno quelli, ne' quali siano necessarj i calcoli algebraici, che non sarebbero stati intesi da molti, capaci per altro d'intendere perfettamente quest'opera. [...] Richiede [la scienza delle acque] senza dubbio ogni regola di prudenza, che il carico dell'esecuzione si commetta più, che ad altri a chi per lungo uso ha potuto osservare, ed apprendere quali facilità, o quali difficultà si sogliono incontrare su i fatti, e come profittando di quelle si possano sfuggire queste con risparmio di danaro, e di tempo, e con vantaggio dello stesso lavoro. [...] ne' libri teorici poco, o nulla d'ordinario si trova scritto né intorno a materiali, né intorno alle manifatture de' lavori [...] sarebbe desiderabile, che alcuno esperto ingegnero desse al pubblico un trattato compito, e metodico sopra tali particolarità [...] concordo che in simili affari siccome a nulla serve una pratica troppo cieca, così resti inutile una teorica troppo astratta e che la perfezione debba consistere in un giudizioso accoppiamento dell'una coll'altra". [Guglielmini 1739, pp. VII-IX].

The purely hydrometric part of Guglielmini's doctrine was accepted as likely by his contemporaries, since certain rules for the velocity of the waters were still missing. As regards Guglielmini's principles about the system of the river beds, they were never questioned; in fact, they had gained faith and good reputation among "the most skilled professors of this art and the engineers who crave to ground their works on valid foundations. They say that this book [Guglielmini's treatise] is an inexhaustible supply of very useful instructions for the regulation of waters". [56]

As regards the job of hydraulic technician, E. Manfredi referred to it using the term "engineer" (*ingegnero*). Some years later the term "civil engineer" (*ingegnero civile*) appeared in the title of a work by Giuseppe Antonio Alberti (1715–1768) that was widely distributed: *Istruzioni pratiche per l'ingegnero civile, o sia perito agrimensore, e perito d'acque*. Alberti noticed that professors should have just one book, that contained all the subjects, divided and scattered in several books, in order to make them clear and plain. Alberti spoke to professors who knew theoretical principles, i.e., geometry, arithmetic, mechanics, perspective and architecture, but were badly instructed about practical rules. Alberti collected much practical information that was useful for the job of civil engineer, i.e land surveyor and hydraulic technician. That information derived partly from his studies, partly from his practical fieldwork, partly from the works by other authors that dealt the same subjects. [57]

The job of the so-called "periti" was "useful, virtuous and highly reputed". It was useful because it supervised important works on the lands and on the waters, on which depended public and private welfare. It was virtuous because it was supported by physical and mathematical doctrines. It was highly reputed because it gained fair salary from its efforts. [58]

The cultural experience that was necessary for practising the job of hydraulic consultant was well described also by B. Zendrini in his treatise.

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56 [Guglielmini 1739, pp. IV-V].

57 "Ho dunque messo insieme un corpo d'istruttive notizie pratiche inservienti all'impiego d'Ingegnero Civile, o sia Perito Agrimensore, e Perito d'Acque, le quali notizie ho cavate, parte da quel poco di studio, che ho fatto, parte dalla Pratica, che mi ha insegnato qualche cosa nell'andare operando, e parte dagli scritti di diversi uomini dotti, ed illustri, che hanno trattato di queste materie". [Alberti 1748, pp. V-VI].

58 "Essa è una professione, che non può essere né più utile, né più virtuosa, né più stimata: utile perché s'impiega nel dirigere importantissime operazioni nei terreni e nelle acque, dalle quali dipende la salute pubblica degli Stati e il comodo particolare d'una infinità di persone private; virtuosa, perché procede colla scorta delle dottrine fisiche e mattematiche; stimata, perché ricava dalle proprie fatiche stipendj onestissimi e ragguardevoli". [Alberti 1748, pp. III-IV].

The hydraulic technicians had to know elementary mathematics, i.e., Euclidean geometry, arithmetic, the principles of algebra ("an arithmetic that was handled with letters and numbers"). As regards mixed mathematics, the technician should know mechanics, that included the doctrine of weights, powers, resistances and equilibrium, both for solids and fluids. [59]

As we have already said before, the fundamental factor of modernization in mathematical teaching in the first decades of the 18th century was the call to the university of Padua of the mathematicians Guglielmini and Hermann, who were deeply involved in the solution of hydraulics problems. Looking a little further, during the 18th century there was an increasing development of the centres for the study of hydraulics, schools, colleges and universities. As regards hydraulics in Lombardy, the figure of the Jesuit Giovanni Antonio Lecchi emerged in Milan in the mid of the 18th century. Even while he carried on his teachings of mathematics and hydraulics at the College of Brera, he mainly worked as a hydraulic engineer and he did many surveys on practical hydraulics. From 1757 Lecchi was interested in technical issues connected to the problems of hydraulic engineering, on behalf of the College of Brera but also of the Hapsburg government. [60] In addition to the colleges, the mathematical teaching took place in the university of Pavia and the Palatine Schools in Milan. Beginning in 1740, a reorganization of the educational system at every level of instruction was carried out by the Austrian Empire. In the autumn of 1763, the government appointed two highly qualified lecturers in Mathematics

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59 "[...] lo studio delle Matematiche elementari, comprendendo sotto di queste la Geometria di Euclide, l'Aritmetica, i principj dell'Analisi, che altro non contengono che un'Aritmetica maneggiata con caratteri e numeri, in vece di servirsi di questi ultimi soli; per altro le quattro operazioni, sopra delle quali si fonda tutta quant'è l'Aritmetica, le stesse e non più servono all'Analisi, e ciò per quello appartiene alla pura contemplazione della quantità discreta e continua. Per le miste Matematiche poscia dovrebbe il Perito ben intendere le meccaniche che comprendono tutta la dottrina de' pesi, delle potenze, delle resistenze e degli equilibrj tanto de' solidi che de' fluidi". [Zendrini 1741, pp. XXIII-XXIV].

60 Giovanni Antonio Lecchi was a professor at the College of Brera where from 1738 to 1760 he taught mathematics, then from 1760 to 1773 he taught mathematics and hydraulics. In his first hydraulic papers, written on behalf of the College of Brera, Lecchi tried to apply mathematics to the problem of the measure of the speed and of the flow rate of water, by using a theoretical approach: *Dissertazione idrostatica di partizione d'acque della Roggia Caccesca per la costruzione de' modelli ne' territorj di Granozzo, e Robbio*, Milan, Marelli, 1743. In 1759 the Austrian Empress Maria Theresa conferred the title of "matematico e idraulico regio" upon him. The hydrodynamic investigations of Lecchi were promoted by R. G. Boscovich and reached their peak with the publication of the *Idrostatica* Lecchi [1765]. On the relationship between Lecchi and Boscovich see Lugaresi [2017c]. Twenty-two letters written by Lecchi to Boscovich in the period 1763–1770 have been transcribed in [Lugaresi 2014, pp. 455–474].

and Astronomy in order to extend and to develop the technical and scientific teachings in Milan and Pavia: P. Frisi and R. G. Boscovich. Frisi was called to teach in the Milan Palatine Schools and Boscovich in the philosophical faculty of Pavia. [61] The lectures of mechanics, hydrometry and theoretical architecture given by Frisi in the Palatine School were published later, *Instituzioni di meccanica, d'idrostatica, d'idrometria e dell'architettura statica, e idraulica ad uso della Regia Scuola eretta in Milano per gli architetti, e per gl'ingegneri* (Milan, Galeazzi, 1777). [62]

In order to give an overview of mathematical education as applied to the motion of waters inside the Italian universities, we will now focus our attention on two particular cases, i.e., the universities of Ferrara and Turin, that were protagonists of two important educational reforms during the 18th century. The city of Ferrara had a long hydraulic tradition in the teaching of hydraulic disciplines, firstly inside the local Jesuit college (1675), then inside the university (1771). In Turin, by contrast, the teaching of hydraulics appeared only during the 18th century.

4. MATHEMATICS AND SCIENCE OF WATERS AT THE UNIVERSITY OF FERRARA

In the mid-17th century specific training for acquiring the skills needed to study waters was required; therefore it was decided that mathematics and hydraulics should have their own particular instruction. At the university of Ferrara, the public lectures of mathematics covered a period of three years and dealt with the first six book by Euclid, the *Sphaera* by Sacrobosco and the planets theory. Among these topics, only the *Elements* by Euclid could be useful for technicians of waters, but practical applications of the theory were still missing. A compromise between public education and the need for studies aimed at hydraulics subjects was represented by the creation of a "private" mathematical lecture in the Jesuit College of Ferrara in 1675. [63] The lectures of mathematics in the Jesuit College had a practical and professionalising purpose, but the theoretical part was also needed.

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61 In the Palatine Schools applied mathematics (the so-called "matematica mista") has been taught, since 1763 when Paolo Frisi introduced the course of mechanics and science of waters. [Brambilla 1990, pp. 359–366]. The scientific figure of Paolo Frisi has been examined in Barbarisi [1990]. The hydraulic work of Ruggiero Giuseppe Boscovich has been studied in Lugaresi [2013].

62 See [Lugaresi 2014, pp. 236–241].

63 We talk about "Private lecture" in order to distinguish it from the "University lecture", even if both of them are public. See Fiocca & Pepe [1985].

The lectures of mathematics in the Jesuit College of Ferrara were intended to prepare the so-called "notai" (notaries) and "giudici d'argine" (judges of river bank), i.e., public technicians in charge of environmental protection. A useful description of this professional figure can be found in the book *L'idea del perfetto giudice d'argine*:

> The judge of river banks was a person who was assigned to the care of river banks. Also under his charge was the care of pipes, canals, ditches, streets, bridges, gutters, underground barrel vaultings, sea harbours, water direction and control. So he had to be very well taught about mathematics, especially arithmetic and geometry. [...] As regards arithmetic, he had to know how to add, to subtract, to multiply, to divide both integer and rational numbers, to extract a square or cubic root, to know the golden ratio and the rule of three. As regards geometry, he had to well know the first six books of the Elements by Euclid, because they enlightened on the knowledge and the practice of their profession. [...] He had to know how to measure surfaces and solids, how to create and read a topographic map. He had also to know the theory and practice of leveling. [...] Briefly, a perfect judge of river banks should be both a good practical and theoretical person and he should be able to account for his conduct. [64]

The study of mathematics in Ferrara developed in close connection with the study of hydraulics. Because of its geography the city throughout its history has always paid a particular attention to advanced scientific techniques in order to solve problems related to controlling river waters, shoring river

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64 "L'Uffitio di Giudice d'Argine ha nella sua denominatione l'espressione delle sue incumbenze, essendo egli destinato a giudicare i bisogni de gli Argini, e successivamente a ripararvi [...] sotto la sua carica sta la cura de condotti, canali, fosse, strade, ponti, chiaviche, ponti canali, botti sotterranee, porti di mare, sbocchi di fiumi e di condotti pubblici, direzione dei fiumi, regolamento delle acque del paese, per lo che gli conviene essere molto bene instrutto e prattico delle Matematiche in universale, e particolarmente dell'Aritmetica, e Geometria. [...] Per quello riguarda l'Aritmetica, dovrebbero saper sommare, sottrare, moltiplicare, e partir di numeri sani, & intieri, e di numeri rotti, & intieri unitamente, e separatamente, schisare, cavare le radici quadrate, e cube, & essere molto prattici della regola Aurea, o del tre. Nella Geometria dovrebbero essere bene esercitati, & havere studiati, & intesi almeno li primi sei libri degl'Elementi d'Euclide, essendo incredibile, quanto lume apportino alla cognitione, e prattica delle cose di loro professione, in tutte le quali, & in ciascuna d'esse sempre o esplicita, o implicitamente molti per ordinario concorrono, ma non basterà già, che habbino imparato, e che siano riusciti buoni Aritmetici, e Geometri, ma sarà di mestieri, che le coltivino, e pratichino, o almeno non ne abbandonino affatto l'esercitio, altrimenti se ne scordaranno in breve; dovrebbero ancora sapere misurare le superfitie, & i corpi solidi, pigliare in pianta i Paesi, e ridurne in carta i dissegni in misura, livellare i siti, e formarne i profili in carta [...] In sintesi, i perfetti Giudici d'argini dovrebbero essere non solo buoni pratici, ma anche teorici, per saper rendere ragione del loro operare". [Lambresagni 1692, pp. 1–8].

banks and surveying land for water control purposes. Since the 16th century a large technical and scientific debate and many hydraulic projects developed around the issue of water regulation in the lower Po valley. Developments in mathematics during the 17th and the 18th centuries led to the emergence of a new mathematical discipline, hydrodynamics, and with it arose the need of a new figure, the hydrostatic mathematician ("Matematico Idrostatico"), to complement the simple technical expert. A plan for the reform of the university was presented in 1771 in order to prepare this new professional figure. The lectures of mathematics provided for a curriculum aimed to prepare high-qualified people for solving the problem of the regulation of waters. There were two different professional figures: the "perito" who had some elementary theoretical knowledge and the hydrostatic mathematician. [65]

An interesting proposal came from Gianfrancesco Malfatti (1731–1807), professor at the university of Ferrara. In the science of waters—as Malfatti said—there were few general theorems. The main part of this subject was still uncertain and obscure so that every town had to take specific exams on the nature of rivers and streams in order to catch the most likely conjectures from direct experience and observations. According to Malfatti, that's why the lectures of mathematics should focus on the rivers of Ferrara province so that students could have direct field experience:

> And since there were few general theorems about the science of waters, the rest remains covered by a disgraceful uncertainty and obscurity so that each Province has to examine the nature of its rivers and streams in order to learn from experiences and observations the more likely conjectures. So, after having dealt with flowing waters and rivers in general, it should examine in depth the rivers of the Ferrarese plain, and students should keep the map at hand in order to know origin, path, fall, mouth of the rivers, waters quality, usual time of their overflows and everything else about the way to well regulate them, to provide them for good embankments &c. [...] It should be illustrated by means of physical and mathematical reasons, by basing itself on the less doubtful theories. [66]

65 See [Fiocca & Pepe 1985, pp. 130–133]. See also Fiocca [2004].

66 "E siccome pochi sono i Teoremi generali della Scienza delle Acque, rimanendo il restante involto in una deplorabile incertezza ed oscurità a segno, che ciascuna Provincia è obbligata, all'occasione, di fare degli esami particolari sulla natura de' suoi Fiumi e Torrenti, per ricavare dalla esperienza e dalla osservazione le più probabili congetture, così, dopo aver trattato delle Acque correnti, e de' Fiumi in generale, dovrebbe più particolarmente estendersi a parlare dei Fiumi del Ferrarese, instruendo i Giovani colle Mappe sotto gli occhi della loro origine, corso, caduta, sbocco, qualità di acque che portano, tempo ordinario delle loro piene, e di tutto ciò, che riguarda la maniera di ben regolarli, munirli di buone arginature &c. [...] illustrando

The training of the hydrostatic mathematician should last four years: the topics were “algebra and mechanics” (plane and solid geometry, trigonometry, Cartesian algebra, conic sections, differential and integral calculus, and mechanics) in the first two years and “hydrostatics and hydrodynamics” in the last two years. In the academic year 1771–72 there was a single lecture course of mathematics (that included mathematics and hydrostatics), held by Gianfrancesco Malfatti. From the academic year 1773–74, there were three chairs, kept by different professors. The chair of algebra and mechanics, held by Malfatti, was responsible for Euclidean geometry, Archimedes’ theorems, trigonometry, conic sections, Cartesian algebra, differential and integral calculus, mechanics. The chair of hydrostatics, held by Teodoro Bonati, covered mathematical formulas for hydrodynamics, general principles for running waters, rivers of the Ferrara province, rules for the drawing of a map and the building of embankments. The lectures of hydrostatics should include some practical exercises in the country, a reason why it should be held by a professor well-experienced with the territory. The chair of practical geometry, held by Ambrogio Baruffaldi, would teach how to use geodetic instruments in order to take measures. [67]

Thus, in the last part of the 18th century inside the reformed university of Ferrara we have a valid educational curriculum for modern hydraulic engineers. Throughout the 19th century mathematics continued to be cultivated and taught at Ferrara as part of a course of studies whose aim was to provide professional training for the engineer.

Teodoro Bonati was a highly-qualified technician in the field of hydraulics. He had wide theoretical knowledge and great skills for problems related with directions of rivers and regulation of waters. In the mid of fifties of the century Bonati was involved, together with his teacher Romualdo Bertaglia in the debate over the regulation of the river Reno. [68]

ogni cosa con ragioni Fisiche e Matematiche, ed appoggiando tutto alle men dubbie Teorie”. [Collectives 1982, p. 10]. Gianfrancesco Malfatti represents one of the most original mathematicians of his time. He was pupil of Vincenzo Riccati in Bologna in the years 1748–1754, then he moved to Ferrara where he was librarian of the Marquis Bevilacqua. In 1771 he became professor at the university of Ferrara. The mathematical works by Malfatti have been published in Malfatti [1981].

67 [Fiocca & Pepe 1985, pp. 133–135]. Ambrogio Baruffaldi was a Ferrarese technician of the Reverenda Camera Apostolica who worked as a judge of river banks during some waters visits.

68 Teodoro Bonati (1726–1820) represented one of the major expert in the field of hydraulics between the 18th and the 19th century. Also Napoleon appreciated him and wanted Bonati as a member of the *Istituto Nazionale.* Bonati cultivated with interest mathematics, thanks to his friendship with Gianfrancesco Malfatti. Bonati was

Bertaglia and Bonati were the main interlocutors for the Ferrarese side, against the city of Bologna, represented at that time by the mathematician Gabriele Manfredi. [69] As member of the Ferrarese delegation, during the sixties of the 18th century Bonati travelled a lot between Ferrara and Rome, where he represented—together with Romualdo Bertaglia—the interests of the city of Ferrara in the debate over the river Reno. [70]

in correspondence with some important members of the Italian scientific community, such as Sebastiano Canterzani, Girolamo Saladini, Paolo Frisi and Anton Maria Lorgna. See Borgato et al. [1992]. These authors published many contributions about hydraulic and hydrostatic themes: Frisi P., *Piano de' lavori da farsi per liberare, e assicurare dalle acque le provincie di Bologna, di Ferrara, e di Ravenna, con varie annotazioni, e riflessioni*, Rome, Stamperia della R. Camera Apostolica, 1760; then Lucca, Giuntini, 1761; Idem, *Del modo di regolare i fiumi, e i torrenti, principalmente del Bolognese, e della Romagna*, Lucca, Giuntini, 1762. Lorgna A. M., *Discorso intorno al riparare dalle inondazioni dell'Adige la città di Verona*, Verona, Moroni, 1768; Idem, *Ricerche intorno alle distribuzioni delle velocità nelle sezioni de' fiumi*, Verona, Moroni, 1771; Idem, *Memorie intorno all'acque correnti*, Verona, Moroni, 1777; Idem, *Discorso intorno al ripararsi dalle corrosioni del Po ne' contorni della città di Piacenza*, Parma, Stamperia Reale, 1778; Idem, *Nuova Teoria intorno al movimento de' navigli a remi*, in "Memorie di Matematica e Fisica della Società Italiana", t. II (1784), s. I, pp. 457–505; Idem, *Teoria Fisico-matematica intorno al moto de' liquidi uscenti da' fori delle conserve*, in "Memorie di Matematica e Fisica della Società Italiana", t. IV (1788), s. I, pp. 369–417; Idem, *Misura dell'impulsione permanente de' liquidi contro le superfici piane*, in "Memorie di Matematica e Fisica della Società Italiana", t. IV (1788), s. I, pp. 418–428; Idem, *Del misurare l'acqua che esce dalle cateratte con moto libero. Memoria I*, in "Memorie di Matematica e Fisica della Società Italiana", t. V (1790), s. I, pp. 313–329; ; Idem, *Del misurare l'acqua che esce dalle cateratte con moto perturbato. Memoria II*, in "Memorie di Matematica e Fisica della Società Italiana", t. V (1790), s. I, pp. 330–355; Idem, *Cateratta idrometrica*, in "Memorie di Matematica e Fisica della Società Italiana", t. V (1790), s. I, pp. 397- 407; Idem, *Legge inseparabile dal principio fondamentale del Castelli intorno al moto e alla misura delle acque correnti*, in "Memorie di Matematica e Fisica della Società Italiana", t. VI (1792), s. I, pp. 218–220. Saladini G., *Memoria idrostatica*, in "Memorie di Matematica e Fisica della Società Italiana", t. XI (1804), s. I, pp. 147–157.

69 Both Manfredi and Bertaglia wrote many polemical papers in order to defend their theses and to criticize those of the rival party: *Esame, e riprova del nuovo progetto di arginare alla destra il Po di Primaro: e proposizione per liberare, e assicurare dalle acque le provincie di Ravenna, di Bologna, e di Ferrara*, Ferrara, Stamperia Camerale, 1758; *Voto del sig. Dottore Gabriello Manfredi sopra il parere de' due periti di Bologna e di Ravenna circa l'arginare il Po di Primaro*, Bologna, 1759; *Esame del voto del signor dottore Gabriello Manfredi, e seconda riprova del progetto di arginare a destra il Po di Primaro. Esposizione, e conferma della proposizione del nuovo fiume per liberare, e assicurare dall'acque le provincie di Ravenna, di Bologna, e di Ferrara*, Ferrara, Stamperia Camerale, 1759; *Risposta del signor Dottore Gabriele Manfredi alla decontro scrittura del Sig. Romualdo Bertaglia*, Bologna, Stamperia Dalla Volpe, 1760; *Replica alla risposta fatta dal signor dottore Gabriello Manfredi all'esame del suo voto di arginare a destra il Po di Primaro: e all'esposizione, e conferma della proposizione del nuovo fiume per liberare, e assicurare dall'acque le provincie di Ravenna, di Bologna, e di Ferrara*, Ferrara, Stamperia Camerale, 1760. See [Lugaresi 2014, pp. 193–205; 213–235].

70 See [Lugaresi 2014, pp. 213–235]. The manuscripts by Teodoro Bonati are collected in twenty-four volumes, preserved in the Ariostea Library of Ferrara. Most of

As a hydrostatical mathematician, Bonati realized many hydraulic experiments on the speed of water on artificial canals, that were described in his work *Sperienze in confutazione delle sperienze del signor Gennetè intorno al corso de' fiumi.* [71]

The importance of practical experiences in the study of the motion of waters clearly emerged in many parts of Italy—we have already quoted the experiences made by Giovanni Poleni in the Republic of Venice in the second decades of the 18th century. But we have to look at Piedmont in order to find an institutional turning point in the theoretical and practical researches in hydraulics.

5. SCIENTIFIC AND TECHNOLOGICAL TRAINING AT THE UNIVERSITY OF TURIN

During the 18th century a general reform of university education was called for in different parts of Italy. [72] In the Kingdom of Sardinia, of which Piedmont was then part, the reform of the university began very early: in 1729 the *Costituzioni Generali per l'Università di Torino* had been issued. The need for reform inside the university of Turin began with the scientific and technological training of the State officials. The university had a central role as bureaucratic organisation that had to check and certify competencies for the exercise of a profession, like that of trainees and apprentices. The faculty of arts included also the chairs of logic and metaphysics, experimental physics, geometry and mathematics. These subjects had also a component of training aiming at professionalization. In 1748 the chair of

these volumes deals with hydraulic themes. A list of these hydraulic manuscripts has been transcribed in [Lugaresi 2014, pp. 325–331].

71 Between 1762 and 1763 Bonati made some experiments in Ferrara, then in Rome in the presence of some other mathematicians like François Jacquier and Thomas Le Sueur. Those experiments allowed Bonati to reject some false theses maintained by Claude Leopold Gennetè (1706–1782). Gennetè made several experiments on river flow that dated back to the second half of the 18th century. These experiments were published in the work *Expériences sur le cours des fleuves*, Paris, Lambert, 1760. The thesis by Gennetè was that if you double the quantity of water in a canal, the height of the water remains the same, being balanced by an increase of speed. Some years later the same experiments were repeated in Rome by Bonati (1762–63) and the final result contradicted that of Gennetè. The Italian translation of Gennetè's paper, together with the work by Bonati, were published in two editions of the collections on the motion of waters. See [Lugaresi 2015, pp. 222–224; 213–232].

72 We have already quoted the teaching reforms that have been carried out in Lombardy (1763) and in Ferrara (1771) in order to promote technical and scientific teachings.

experimental physics had been created at the university of Turin and it had been entrusted to Giambattista Beccaria (1716–1781). [73]

When Beccaria was appointed to the chair of physics in Turin, he introduced the experimental method (in contrast with the Cartesianism of his predecessors). He kept the chair for more than thirty years. Experimental activity was characteristic of Beccaria's scientific work. Besides educational tasks, Beccaria took part in different projects related to his knowledge of mechanics, astronomy and hydraulics: the revision of the system of weights and measures of the kingdom of Sardinia, the placement of lightning rods, the measure of the meridian arc in Turin and the evaluation of a unit for the distribution of the water of the river Po. Most of these topics appeared in Beccaria's correspondence with Ruggiero Giuseppe Boscovich. Beccaria had met Boscovich during his stay in Rome in 1744, then they kept in touch by correspondence. [74] Among the subjects discussed by the two correspondents were the measure of the meridian arc in Piedmont and the studies for a unit of measurement for waters. These themes were very well known to Boscovich who was a reliable advisor for Beccaria. In the mid of the 18th century Boscovich was one of the most appreciated consultants for problems of applied mathematics. Since 1751 he was often asked for the solution of hydraulic problems and was one of the most significant figures in the study of the motion of waters in the second half of the 18th century in Italy. [75] Beccaria too was extensively involved in the research field of the science of waters. [76]

During the 18th century the policy of the Savoy government undertook many administrative and legislative reforms that culminated in the promulgation of the *Leggi e costituzioni di Sua Maestà* (1723, then a second version

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73 On the figure of Giambattista Beccaria see Vendola [2000].

74 The correspondence between Boscovich and Beccaria covered the period from 1755 to 1770. See Manara [to appear].

75 As a consultant on problems relating to the science of waters, Boscovich was asked by many Italian courts for consultations. He wrote reports regarding the regulation of some rivers and streams, some harbours on the Tyrrhenian and the Adriatic Sea, the reclamation of large marshlands. His main contributions dealt with the settlement of harbours placed at the mouth of river. Boscovich's hydraulic works have been published in Lugaresi [2013].

76 Beccaria's hydraulic manuscripts were collected by his pupil Prospero Balbo (1762–1837) and even now are kept in the Apostolic Vatican Library inside the *Fondo Patetta*. A reconnaissance of this archive has been done by L. Moscati. The manuscripts are kept in the folder, entitled *Misura delle acque*. They can be divided in four parts: some papers related to the *Memoria* written by Beccaria on behalf of Carlo Emanuele III; some practical experiments on waters made by Beccaria; some other preparatory papers about the measure of waters; many drafts and sheets of drawings, sketches and calculations. See [Moscati 1990, pp. 489–490].

in 1729). With this measure the King Vittorio Amedeo II extended the concept of "state-owned asset" (*bene demaniale*) to the waters. These laws had to strengthen royal and fiscal rights, but also to carry out a large distribution of public waters in the public interest and a fair use of water transfers. The King assumed control of the waters, supporting the profits of irrigation in the Piedmontese countryside, in order to counter the private appropriation of water. It was necessary to take advantage of royal intendants in order to provide surveys about a fair measure of waters. After the laws of 1729, there was an increasing number of edicts, decrees and licenses for the concession of the opening of new canals or derivation of water, but a general program was still lacking. [77]

The topic of hydraulics at the service of agriculture and irrigation has been marginal for a long time in the scientific and technical debate. Since Italian unification, only in the northern Italy there was a wide net of irrigation canals and advanced systems for the water measure and distribution. The Duchy of Milan, where there were the greatest Italian water sources, developed the most technologically advanced systems. As regards water measure and distribution, the method that had been developed since the Medieval period, was called "method of the ounce of water". It consisted in evaluating the quantity of water that came out in a given time from a mouth of given shape and dimensions in order to guarantee that the distributed volumes were multiples or submultiples of the local measure. During the 18th century in the Milanese territory there was a wide debate about a reform of the systems in use in Italy for water measure and distribution. [78]

Because an established rule was lacking, many systems were adopted in Piedmont for water distribution, so that there were a lot of variations and contradictions. In order to solve this situation, since 1763 the King Carlo Emanuele III promoted a survey that dealt also with the waters. Among the orders of the cadastral census, the King asked for information about measures concerning the recording for the beds of rivers, streams and irrigation canals. The king appointed many experts, who were "suitable to do

77 See [Moscati 1990, pp. 492–494].

78 In the debate two ideas were counterposed: a moderate reformism, supported by G. A. Lecchi, and a more radical position, supported by P. Frisi. Lecchi suggested to bypass the problem of different local modules of the ounce of water by identifying a rectangular under a head module through direct measure of the flow rate in the canal. The problem of the reform of the systems for water measure and distribution, so important for Italy, remained open. The debate came up again some decades later and the main interlocutors were Vincenzo Brunacci and Antonio Tadini. See Di Fidio & Gandolfi [2014].

the most diligent studies and the greatest experiences" (*idonei di fare i più diligenti studj, e le più grandiose sperienze*), in order to search for "the best possible way to distribute waters with right and clear measure, so that by distributing waters in a more exact proportion it could be more widely promoted the fertility of the farmland, justice could be better managed [...] and it could be put to an end every uncertainty, every arbitrary act of the consultants and every arising quarrel". [79]

During the execution of the census survey, in 1764 the King appointed Beccaria to determine a unique modulus for the measure of the waters. This problem was connected with agriculture and aimed to find a unique unit of measurement for the distribution of waters. Beccaria together with his university assistant Domenico Canonica (1739–1790) was involved in this work and made some experiences in a farmhouse near the real park of the Venaria (*Cassina dei Merli*) in Turin. Thanks to Beccaria's study, the ounce of water was defined as unit of measurement for the waters. The results had been exposed and approved by a royal hydraulic Commission, whose members were Antonio Felice De Vincenti (head of the Royal Unit of Artillery), Alessandro Vittorio Papacino d'Antoni (general director of the Royal Schools of Artillery and Fortification) and Francesco Mattei (engine driver of the Royal Arsenal). Beccaria was charged with writing an essay in order to establish a legislation for the measure of waters. [80]

Some references to the evaluation of a unit for the distribution of the waters can be found in the correspondence between Boscovich and Beccaria for the year 1764. In order to implement his theories about waters, Beccaria made some practical experiences by means of the siphon, trying to adapt this device to a very wild canal ("rivo incostantissimo"). [81] Beccaria complained for the difficulties in defining an invariable unit for the measure of the waters, so that it was almost impossible in many cases. Boscovich

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79 "l'ottima possibile maniera di distribuire le acque con giusta ed evidente misura; sicché distribuendo le acque in proporzione più esatta si promovesse più ampiamente la fertilità medesima delle campagne, si potesse amministrare convenientemente la giustizia [...] e si ponesse fine a tutte le incertezze, ai ciechi arbitri de' periti, ed a tutte le liti, che ne insorgevano". [Moscati 1990, p. 497].

80 Beccaria G. B., *Memoria intorno alla possibilità di una legislazione intorno alla misura dell'acqua sufficientemente esatta e comunemente intellegibile.* See [Moscati 1990, pp. 498–500]. Only in 1837 the Albertine Code would establish the ounce of water as unit of measurement, gathering the different systems existing until then in the different parts of the Kingdom of Sardinia. See [Moscati 1990, p. 521].

81 [Lugaresi 2017a, p. 13]. The method of the siphon was also noted by the Royal Commission. The hydraulic experiments were realized by Beccaria during the years 1764 and 1765. See [Moscati 1990, pp. 501–505].

referred some experiences he made in Rimini during an inspection in the city in order to solve the problem of its harbour. [82]

Beccaria's hydraulic experimentation was contemporary to that of Francesco Domenico Michelotti (1710–1787) inside the so-called "Stabilimento della Parella". The laboratory of Parella was a structure for hydraulic experiments, created in order to study the motion of water. It can be considered one of the most representative examples of the union between educational and research structure. The building work was patronised and financially sponsored by the King Carlo Emanuele III. [83] The building was located not far from Turin, just outside the Porta Susa. It consisted of an empty square cross-section tower where water entered from the top and came out from the bottom through holes of different shapes put at different heights, which coincided with the three floors of the tower. [84]

Michelotti, who was the main promoter of the creation of the laboratory of Parella, was professor of mathematics in the university of Turin. He was very well skilled both in pure and applied mathematics, especially as regards hydraulics. Two were the fundamental principles of hydraulic science: velocity proportional to the square root of the height of falling and reciprocity of the sections with their average velocities in the hypothesis of stable and continuous flow (Castelli's law). These two principles were sufficient for theory, which set aside resistances, but not for practice because nature didn't set aside resistances. So, "we have to look for a third principle in favour of practice, i.e., the law of hydraulic resistances". As it was too difficult to find this law just with theoretical principles, it was necessary to turn to practical experience. In past times Newton, Daniel Bernoulli and Poleni found something but they couldn't develop it because they could do only few experiments and "on a smaller scale, though with the utmost care". The check of this law needed a great number of various experiences. That's why the laboratory of Parella was created. Several experiences, made during many years, allowed to demonstrate with facts "the combination of the three principles in the motion of waters". [85]

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82 See [Lugaresi 2013, pp. 233–290].

83 In Maffioli [2015] the author compared the hydraulic experiences in Turin with those made in Padua by Giovanni Poleni in the first decades of the 18th century. He observed the experimentation of the Venetian mathematician was philosophical and private, while Michelottis's one had a public and institutional relevance. See [Maffioli 2015, pp. 92–97].

84 The laboratory of Parella had been studied in [Lugaresi 2017a, pp. 130–141]. Francesco Domenico Michelotti was appointed professor of mathematics at the university of Turin in 1754 and he kept the chair until his death. See [Lugaresi 2017a, pp. 13–16].

85 Michelotti [1767–1771], vol. 1, Preface.

Michelotti remarked that "from the combination of the three principles in the motion of waters, we can get a very easy method for determining both velocity and flow just with the simple practical geometry and ordinary arithmetic". The best period to make hydraulic experiences was between the end of summer and the beginning of autumn, when the water of the river Dora flowed clear. The observations, made between 1764 and 1766, were collected and described in the volume entitled *Sperimenti idraulici principalmente diretti a confermare la teorica, e facilitare la pratica del misurare le acque correnti*. The volume was divided into three parts: the first one included many experiments about water that came out from three different heights through holes of different shape and dimension, provided with funnel, pipe or both of them. The description of hydraulic experiments was followed by a theoretical proof of the first principle of the science of waters. The second part of the volume dealt with water in progressive motion and the proof of the third principle, i.e., the law of water resistances. Michelotti explained the practical rule for determining velocity and flow of water in regular canals, confirmed by many experiments, and he mentioned the way to adapt it to irregular canals. [86]

The volume, besides containing a great quantity of hydraulic experiences, provided useful details in order to understand how water experiments were made and what skills the technicians who took part in them required. The required mathematical skills were rather "elementary": a good knowledge of the rules of arithmetic calculation, both with rational and irrational numbers, the theory of proportions, plane and solid geometry, necessary for the calculus of areas and volumes, the trigonometry. As the experiences were subject to measurement or approximation errors, Michelotti stressed many times that "In such numerous experiments one cannot hope for or demand geometric accuracy". [87]

Soon after the publication of his work (1767), Michelotti sent a copy of it to the most skilled mathematicians in the field of the science of waters. The work of Michelotti was favourably accepted by the Italian scientific community of his time, even if it gave rise to some doubts. R. G. Boscovich and Teodoro Bonati, two of the of the main experts in the field of hydraulics, took part in the debate.

86 Michelotti described several instruments and their use for practical measure of the velocity in the canals: the floating hydraulic speedometer, the paddle wheel, the Pitot tube, the regulator of Guglielmini, the syphon, the quadrant. [Michelotti 1767–1771, vol. 1, pp. 136–168].

87 [Michelotti 1767–1771, vol. 1, p. 72]. In Michelotti's treatise the new instruments of infinitesimal calculus do not intervene.

Michelotti had a brief but rich scientific correspondence with Bonati. The Ferrarese mathematician appreciated Michelotti's work but at the same time he criticised some of his experiments because they did not conform to those Bonati made in Ferrara. The correspondence covered the period between the publication of the two volumes by Michelotti. The first letter was sent by Bonati on November 3rd 1768 in order to thank Michelotti for the sending of his first volume of the *Sperimenti idraulici.* Bonati appreciated the experiences on the waters made by Michelotti in Turin.

> My inclination for physical experiences, particularly those about waters, gave me courage to explain you some of my thoughts about your really nice experiments, that you had exposed with all the clarity and precision. Excuse me if I gain such confidence, absolutely not to contradict, but just in order to acquire more enlightenment in a subject that is very interesting for the state. [88]

In the letters the two correspondents talked about hydraulic experiments. Bonati realized some experiences during an inspection on some diversion canals of the river Adige and compared them with the results obtained by Michelotti in Turin. The last letter of the correspondence dated back to January 15th 1772. Michelotti wrote to Bonati and announced him the publication of a second volume of *Sperimenti idraulici.* Michelotti had planned a second volume of his work in order to answer to the request for making new hydraulic experiences and correcting some other ones. The second volume, published in 1771, was divided into three parts. The first and the second parts were devoted to hydraulics, while the third one was purely analytical. [89] In the second part Michelotti answered the doubts of Bonati about some experiences for water measure. [90] He pointed out that the corrections and the advice given by Bonati in the past letters had been very useful for him to resolve many doubts. [91]

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88 "L'inclinazion mia per le sperienze fisiche, particolarmente in materia d'acque, mi fa coraggio ad esporle alcuni miei pensieri intorno agli sperimenti di V.S.Ill.ma veramente bellissimi, ed esposti con tutta la nettezza, e precisione. Soffra di grazia in pace se io mi prendo questa confidenza non mai per contraddire, ma per acquistare maggiori lumi in una materia molto interessante per il Paese". The correspondence between Michelotti and Bonati is kept in the Ariostea Library of Ferrara, among the manuscripts of Teodoro Bonati. The letters have been transcribed in Lugaresi [2017a].

89 In the third part there were two mathematical papers: the first one was about geometrical progressions and proportions, the second one dealt with irreducible cubic equations. Both the papers were written in Latin twenty years before. Michelotti translated them in Italian in order to publish them at the end of the second volume of the *Sperimenti idraulici.*

90 [Michelotti 1767–1771, vol. 2, pp. 101–115].

91 Some years later some more references to the correspondence between Bonati

The laboratory of the Parella was not only a research laboratory, but had also educational goals. Soon after the publication of the *Manifesto del Magistrato della Riforma riguardante gli studi, esami ed esercizio per le professioni di agrimensore, misuratore, architetto civile ed idraulico* (1762), a functional structure for the practical teaching of hydraulics was required. [92] The university of Turin issued new rules for practical and technical jobs. The land surveyors and the measurers needed respectively a training period of two and three years. Architects instead were technicians who required a preliminary theoretical knowledge in order to practice their job, so they had to attend a specific university course. Civil architects had to study geometry and mechanics, while hydraulic architects had to complete the whole mathematical course, starting from geometry. [93]

Thanks to the laboratory of the Parella, the university could provide a suitable knowledge to future technicians. Michelotti and his successors in the chair of mathematics at the university had a double role, not only as professors at the university but also as supervisors of the hydraulic laboratory both for teaching and research activities. [94]

6. CONCLUSIONS

As regards the science of waters, theory and practice were complementary. Hydraulics is both a mathematical and applied science. This double significance is confirmed by the use of university professors of mathematics as experts in the science of waters and consultants at the service of the

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and Michelotti could be found in Bonati [1784]. In this paper Bonati referred his debate with Michelotti about the use of hydraulic instruments for the measure of waters like Pitot tube.

92 These laws would be improved in the *Costituzioni per l'Università di Torino* (1772). Another reform of mathematical studies was due to Ignazio Michelotti, one of the sons of Francesco Domenico, who succeeded to his father in the direction of the laboratory at Parella: Michelotti I., *Piano di studi matematici* (1799). The document has been transcribed in Lugaresi [2017a].

93 From a professional point of view, the jobs of architect and engineer had many similarities, but from an institutional point of view they were different. The engineer had a military education and career and he belonged to a unit at the exclusive service of the State. The architect was an independent contractor whose main referent was the civil society, so private people or public customer.

94 During the twenties of the 19th century some important decisions, taken by the professors of scientific faculties of the Turin university, began to promote some improvements that supported research and teaching. A new generation of applied scientists was going to be gradually developed: first of all, Giorgio Bidone and his pupils Carlo Ignazio Giulio, Ignazio Pollone and Luigi Federico Menabrea. [Roero 2013, pp. 368–370].

State. Starting from the 18th century the chairs of mathematics developed greatly and they were held by professors with a particular interest for applied hydraulics.

The importance of hydraulic experimentation began to emerge also outside in Italy. In the second half of the 18th century, hydraulic experimentation was promoted in France by the Minister Turgot and dealt with some studies about the canal of Picardy. Since the seventies of the century, the limits of the theoretical approach to hydrodynamics became clear. In a letter to the Minister Turgot, Condorcet underlined that the theory of the motion of fluids was still little advanced in order to solve general problems, so it was necessary to turn to practical experiences:

> [...] Mais la théorie des fluides est trop peu avancée pour résoudre le problème général, et dans chaque cas particulier, il ne peut être bien résolu qu'après avoir fait des expériences. Tout méchanicien qui s'est occupé d'hydraulique n'est pas en état de le faire, il faut dans ce genre un homme qui réunisse la théorie à la pratique, et qui soit en état de décomposer ces problèmes qui sont très compliqués, d'examiner par la voie de l'expérience chaque cause séparément, et M. l'abbé Bossut a fait sur l'hydraulique un livre à la fois très savant et plein d'expériences. S'il y en avait quelques-unes à faire ce serait lui qu'il faudrait choisir. Je lui ai parlé de cet objet, il sera prêt à aller au canal de Picardie quand il plaira à M. le Controlleur général, mais il observe que si l'on juge à propos de décider cette année, il n'y a pas de temps à perdre parce que le beau temps est nécessaire à ces expériences. Au reste il regarde ce voyage non seulement comme utile au canal de Picardie mais comme pouvant le devenir à l'hydraulique en général. Cette science a été peu cultivée en France. Maintenant on l'a abandonnée aux ingénieurs des ponts et chaussées, en Italie au contraire elle a été confiée constamment aux savans les plus célèbres. [95]

In March 1775 D'Alembert, Condorcet and Bossut were appointed scientific members of a specific state commission (*Commission de navigation intérieure*) in order to supervise theoretical and practical experiences on the canals. The three mathematicians made some experiments at the École militaire (July 1775), that were later described in the work entitled *Nouvelles expériences sur la résistance des fluides* Bossut [1777]. French hydraulic experiments shared some aspects with those made in Turin. Both of them were promoted and financed by the State. Besides it, French and Italian hydraulicians tried to realise experiences that were the most similar to practical conditions, i.e., on a large scale. So they had to be as accurate and verified as possible.

95 Lettre de Condorcet à Turgot, été 1774. See [Bru & Crépel 1994, p. 94].

> [...] dans les matières de physique, où l'on cherche seulement à découvrir la marche générale d'un phénomène sans la vouloir soumettre à la précision du calcul, les expériences doivent être faites aussi en grand qu'il est possible. Mais veut-on obtenir des résultats précis et destinés à jeter les fondements d'une théorie? Les expériences fort en grand n'ont plus le même avantage. Presque jamais elles ne peuvent être assez exactes, assez répétées, assez variées, pour faire disparaître les différences sensibles qui se trouvent entre plusieurs observations semblables. Entre ces deux écueils, il y a un milieu à prendre. Opérer assez en grand pour rendre les effets distincts, sans sortir des bornes compatibles avec la précision : voilà le principe qui doit diriger les expériences auxquelles on veut appliquer la Géométrie. [96]

In his experiments on fluid resistance Bossut used some models of ships of different shape and dimension. These boats, loaded with different weights, moved in the basin of the École militaire of Paris. Their times of movement were measured with "an excellent pendulum". Each experiment was repeated and verified many times. In its organizational structure, this kind of experiences on a large scale were similar to the experiments on flow made by Michelotti in Turin.

During the 19th century, in the study of the motion of water the historical approach, based on a critical and detailed reading of old reports and observations, gradually lost interest to the advantage of theoretical explanation and more refined measuring instruments. So the path undertaken in hydraulic research was oriented towards the improvement of observation tools. Among the devices for measuring the flow rate that were developed and improved since the end of the 18th century there were the so-called "Venturimetro" (Venturi meter') or the "mulinello di Woltman" (Woltman meter). [97]

Michelotti's experiences were continued and completed by his sons Giuseppe Teresio (1762–1819) and Ignazio (1764–1846). [98] Some years later, Giorgio Bidone (1781–1839), pupil of Ignazio Michelotti, who was

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96 [Bossut 1777, pp. 5–6].

97 The Venturi meter was a device that allowed the measurement of the flow rates of the fluids in the pipes under pressure. The instrument owed its name to the Italian physicist Giovanni Battista Venturi (1746–1822) who described it in the work *Recherches expérimentales sur le principe de communication latérale dans les fluids appliqué à l'explication de différens phenomènes hydrauliques* (1797). The Woltman meter was a propeller water speed meter. It was invented in 1790 by the German engineer Reinhard Woltmann (1757–1837).

98 The laboratory of hydraulics of Turin was appreciated also outside Italy. Jean Nicholas Hachette (1769–1834) gave a description of this laboratory and he also suggested to create a similar structure in France, near Paris: " Il serait à desirer qu'on profitât des eaux du canal du l'Ourcq, pour former près de Paris un observatoire

charged with the direction of the laboratory of Parella, realised several experiments on the waters. Bidone's studies dealt with weirs and orifices. He experimentally studied the phenomenon of jet contraction and determined the discharge coefficient. His observations and measurements are still valid nowadays. [99]

In Bologna, Giuseppe Venturoli, professor of applied mathematics, mechanics and hydraulics at the local university, inspired by the reading of the *Mécanique analytique* by Lagrange, in particular the section *Hydrodynamique*, began to deal with the study of the motion of fluids from a theoretical point of view. [100] Since the twenties of the nineteenth century, the road opened by Venturoli towards hydraulic studies was carried on by many Italian scientists like Gabrio Piola (1794–1850), Pietro Paleocapa (1788–1869), Elia Lombardini (1794–1878), Carlo Ignazio Giulio (1803–1859), Domenico Turazza (1813–1892), Francesco Brioschi (1824–1897). In his treatise about hydrometry for engineers, Turazza reiterated that "hydraulics was a purely experimental discipline that had to give to the calculus only the task of writing summarily the rules proved by experience". Turazza's work should be a treatise about "applicable experimental hydraulics rather than inapplicable rational hydraulics". [101] Also Francesco Brioschi confirmed this point of view. As regards the role of mathematics in hydraulic researches, Brioschi acknowledged the importance of the latest experiences made in France, German and U.S.A. to discover the law of the efflux and of the motion of water inside pipes and canals, following the "good tradition of Italian hydraulic school, starting from Torricelli and Guglielmini and nowadays worthily represented by Elia Lombardini". However,

> Mathematics can have a double function in hydraulics. A given principle of rational mechanics is clearly applicable to the equilibrium or to the motion of water and, in such case, mathematics serves to deduce from that principle all the

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semblable à la Parella de Turin. Cet établissement, mis sous la direction de la Société Royale d'Agriculture, contribuerait puissamment aux progrès d'une science, l'Hydraulique, dont les applications dans un pays agricole tel que la France, deviendraient une nouvelle source de richesse". See [Hachette 1819, p. 255].

99 The figure of Giorgio Bidone played an important role in science in Piedmont in the first half of the nineteenth century. See Lugaresi [2017a]. The work provided a framework of mathematical sciences in Turin in the period from 1766 (the year of Lagrange's departure from Turin) to the end of the 1830s. This research was mainly based on documents from the Archive Collection of Giorgio Bidone, conserved in the Central Library of the Polytechnic University of Milan.

100 See Venturoli [1806–1807].

101 See [Turazza 1867, pp. VI-VII].

consequences required by the practice. If such principle is lacking, we entrust to the simple observation the solution of an hydraulic item and mathematics gives tools in order to discover the general law. [102]

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102 "Le matematiche ponno avere nell'idraulica una duplice funzione; o un dato principio di meccanica razionale è evidentemente applicabile all'equilibrio od al movimento dell'acqua, ed in questo caso le matematiche servono a dedurre da quel principio tutte le conseguenze di cui la pratica abbisogna; od in difetto di quel principio affidiamo alla sola osservazione la soluzione di una quistione idraulica, e le matematiche offrono mezzi per risalire, da una serie di risultati dovuti ad essa, alla legge generale che comprende quella soluzione". [Brioschi 1866, pp. 102–103].

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