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ÉDITORIAL

Sciences en danger, revues en lutte

Depuis le début de l'année 2020, plus d'une centaine de revues académiques se déclarent «en lutte». Leurs comités de rédaction protestent en particulier contre les propositions contenues dans les rapports pour la loi de programmation pluriannuelle de la recherche (LPPR). Par son ampleur et par sa forme, cette mobilisation est historiquement inédite. La dynamique collective qu'elle suscite, par-delà les disciplines, les écoles et les conditions d'exercice de chacune des revues, témoigne du sentiment de révolte que provoquent ces réformes. La LPPR ne fera qu'aggraver le manque de moyens, de postes et de stabilité, et approfondir les inégalités qui minent l'enseignement supérieur et la recherche, et que deux décennies de «réformes» massivement contestées n'ont cessé d'amplifier.

Dans ce contexte, nos revues scientifiques occupent une place singulière et paradoxale. Lieux d'un intense travail collectif de production et supports efficaces de diffusion des savoirs, elles tendent à être instrumentalisées et mises au service de la vision néo-managériale dominante de la recherche, en particulier par l'utilisation croissante d'indicateurs bibliométriques dont les limites ont pourtant été largement documentés. Si cette économie de la connaissance assure l'enrichissement du savoir, elle rapporte toutefois peu en termes financiers. Elle est en effet adossée à une infrastructure invisible, celle du service public de la recherche.

C'est pourquoi, en refusant de nous tenir à distance de ce qui se joue dans la communauté scientifique comme dans le monde social, nous souhaitons mettre en avant aussi bien *ce* qui fait les revues que *celles et ceux* qui les font. Car notre travail collectif, intellectuel et éditorial, qui permet la production et le partage des savoirs, est directement menacé par les projets de loi actuels, qui fragilisent toujours plus le service public de l'enseignement supérieur et de la recherche.

Parce que le service public en général, et celui de la recherche en particulier, sont menacés, nous, collectif des revues en lutte, nous opposons aux projets de réforme en cours avec la plus grande fermeté.

Le comité de rédaction international de la Revue d'histoire des mathématiques a décidé à l'unanimité de rejoindre le mouvement collectif des «revues en lutte» [1].

La rédaction

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[1] On trouvera l'éditorial commun complet (en plusieurs langues) adopté par ces revues à l'adresse `https://universiteouverte.org/2020/03/05/edito-commun/`.

EDITORIAL

Why French Academic Journals are Protesting

Since the beginning of 2020, over a hundred academic journals have announced that they have "joined the struggle". Their editorial boards have teamed up with the ongoing social movement protest, in particular against the upcoming law for the pluriannual programming of research known as LPPR. In its extent and its form, this is a historically unprecedented mobilization. A collective dynamic has swept the community, beyond disciplinary boundaries, schools and the working conditions of individual journals, reflecting the widespread uproar sparked by these reforms. The LPPR research law will only make the lack of resources, of positions and of stability worse, and deepen the inequalities that undermine higher education and research, amplified by two decades of massively contested reforms.

Against this backdrop, our scientific journals play a distinctive and paradoxical role. They are venues where intense collective, productive work is performed, as well as effective outlets for the dissemination of scholarship, but they tend to be instrumentalized and mobilized in support of a dominant neo-managerial vision of research, in particular through the increasing use of bibliometry, whose limits have been abundantly documented. Though the economy ensures an enrichment of knowledge, it offers little in the way of financial profit, and its survival relies on an invisible infrastructure - that of the public service of research.

This is why, by refusing to distance ourselves from events affecting the scientific community and the social world beyond, we intend to highlight what our journals make happen, as well as the men and women who make *them* happen. Indeed, our collective intellectual and editorial work, which allows the production and sharing of knowledge, is directly threatened by currently pending bills, which are slated to further weaken the public service of higher education and research.

As public service in general, and the public service of research in particular, are being put at risk, we, the collective of journals joined in struggle, are taking a firm stand against the current reform projects.

The international editorial board of the Revue d'histoire des mathématiques has thus unanimously decided to join the collective movement of "journals in struggle". [2]

The Editors

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2 The complete text we are supporting is to be found in several languages at https://universiteouverte.org/2020/03/05/edito-commun/.

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NUMBER THEORY IN THE *NOUVELLES ANNALES DE MATHÉMATIQUES* (1842–1927): A CASE STUDY ABOUT MATHEMATICAL JOURNALS FOR TEACHERS AND STUDENTS

JENNY BOUCARD

ABSTRACT. — The *Nouvelles annales de mathématiques* were a French mathematical journal, published between 1842 and 1927, intended for teachers and students. In this paper, I rely on a systematic analysis in the *NAM* of number theory content—a mathematical field that was virtually absent from French education programs during the period under consideration here. By articulating quantitative and qualitative approaches, my goal is twofold: to study what specific forms number theory takes through this specific *media* on the one hand; and to question the specific character of the functioning of a journal like the *NAM* in the case of number theory on the other. For this, I take into account the actors involved (editors, authors, readers), the various forms of texts published, and the themes studied, as well as the arithmetical practices and discourses that are brought to bear.

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Mots clefs. — Journaux mathématiques, théorie des nombres, circulation des mathématiques, histoire de l'enseignement.

A preliminary version of this paper was presented under the general topic "Journaux et revues destinés aux enseignants et/ou consacrés à l'enseignement des mathématiques" proposed by Livia Maria Giacardi and Erika Luciano during the Fourth International Conference on the History of Mathematics Education in Torino (2014) and I want to thank them. I am also very grateful to the anonymous reviewers of this article for their constructive comments and to Tom Archibald who read the latest English version.

RÉSUMÉ (Une étude de cas sur les journaux mathématiques pour enseignants et élèves : la théorie des nombres dans les *Nouvelles annales de mathématiques* (1842–1927))

La revue *Nouvelles annales de mathématiques* était un journal mathématique français, publié entre 1842 et 1927 à destination des enseignants et des élèves. Cet article repose sur l'analyse systématique dans les *NAM* des contenus de théorie des nombres — domaine quasiment absent des programmes d'enseignement français pendant la période considérée. En articulant approches quantitative et qualitative, mon objectif est double : étudier quelles formes spécifiques prend la théorie des nombres à travers ce *media* spécifique d'une part ; interroger la spécificité du fonctionnement d'un journal comme les *NAM* dans le cas de la théorie des nombres d'autre part. Pour cela, je m'appuie sur les acteurs impliqués (rédacteurs, auteurs, lecteurs), la diversité des formes éditoriales en jeu, les thématiques étudiées ainsi que les pratiques arithmétiques et les discours mobilisés.

1. INTRODUCTION—WHY STUDY NUMBER THEORY IN THE *NOUVELLES ANNALES DE MATHÉMATIQUES*?

The growth of periodicals containing mathematics since the 18th century has participated to the « restructuring of the mathematical communication system » [Gispert et al. 2014]. [1] Since the 1840s, the rapid development of an « intermediate press » [Ortiz 1994], i.e., journals for teachers and students, contributed to this restructuration. These journals offered mathematical content that was related to education or that was considered as « elementary », and therefore accessible to the target readership. They contained multiple editorial forms, from original articles to reviews of textbooks, descriptions of the contents of the curricula of teaching institutions, and mathematical questions intended for the training of the candidates. These intermediate journals were used by various authors. For example, after the *Comptes rendus hebdomadaires de l'académie des sciences* (*CRAS*), the intermediate journal *Nouvelles annales de mathématiques* was the principal

1 Since the 1990's, several works on history of mathematics have been devoted to the study of journals or have taken into account the specific form of mathematical journals in the analysis of a mathematical topic. For example, the scientific project Cirmath, funded by the *Agence nationale de recherche* between 2014 and 2019 and led by Hélène Gispert, Philippe Nabonnand and Jeanne Peiffer, studies the circulation of mathematics, especially through journals, over a long period of time (1700–1950). Two thematic volumes were recently published in this context: « Échanges et circulations mathématique. Études de cas (18^{e}-20^{e} siècles) », *Philosophia scientiæ* (19(2), 2015) and « Interplay Between Mathematical Journals on Various Scales, 1850–1950 », *Historia Mathematica* 45(4), 2018.

journal where the members of the *Société mathématique de France* (*SMF*) published mathematics [Gispert 2015].

The aim of this paper is to study a mathematical domain—number theory—through the systematic analysis of a journal of mathematics for teachers and students—the *Nouvelles annales de mathématiques* (*NAM*)—in order to identify the various forms taken by number theory in this specific context. [2] The *NAM* is here used as a valuable observation point in order to study the means of circulation of number theory in a *milieu* linked a priori to mathematical teaching, by articulating quantitative et qualitative approaches. Reciprocally, the results will be used to question if a teaching mathematical journal such as the *NAM* operates in a specific way through the number-theoretic lens.

The *NAM* was published between 1842 and 1927, in 84 volumes with approximately 11370 contributions by 1860 identified authors. [3] It thus constitutes a relevant way to analyse mathematical content for a so-called intermediate public over a relatively long time. Its explicit target readership was students who were preparing for the admission examination of the *École polytechnique* and the *École normale supérieure* (*ENS*). From 1888, students studying for *licence* and *agrégation* were also mentioned as expected readership in the journal. [4] The *NAM* had a unique position in the French publishing landscape for the first 25 years of its existence. However, from 1877, the growth of the number of students spurred the creation of other periodicals with the same aims, in France and abroad. With the multiple evolutions of mathematical, institutional and editorial contexts, the mathematical content, the authorship and readership of the *NAM*, and the associated dynamic of circulations changed sensibly during its publishing period. The

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2 All the *NAM* volumes are digitised and available at the following address: `http://www.numdam.org/journals/NAM/`

3 By identified authors, I mean authors who signed their texts even if I could not identify their profession or their birth date for example.

4 The *Licence* is one of the degrees awarded by the *Université* in France and the *agrégation* is the competitive examination by which French teachers were recruited. It is difficult to estimate the number of potential student readers of a journal like the *NAM* over the entire period considered, but some quantitative data are nevertheless available in the secondary literature. The number of students in preparatory classes increased throughout the 19th century [Nabonnand & Rollet 2013, 1] and Bruno Belhoste estimated that there were at least 700 students in *mathématiques spéciales* in public preparatory education in 1843 [Belhoste 2001]. In addition, between 1875 and 1913, the number of students was just over 500 per year at the *École polytechnique*, between 500 and 800 at the *École centrale*, between 40 and 70 at the *ENS* and between 120 and 5800 (following major university reforms: see below) in the science faculties [Lundgreen 1980, 328–329].

choice of the *NAM* is also based on the existence of a collective research work engaged in a global study of the *NAM* and led by Hélène Gispert, Philippe Nabonnand and Laurent Rollet, which permits us to put the results obtained for a specific domain in perspective. [5]

Number theory represents an interesting case study for several reasons. First, number theory had a marginal position in the mathematical literature during the period considered here: as an example, between 1870 and 1914, the average proportion of pages on number theory in the German reviewing journal *Jahrbuch über die Fortschritte der Mathematik* was between 3.5 and 4% [Goldstein 1999, p. 196]. In a certain way, this marginality allows us to develop a systematic study of every occurrence of number theory because the corresponding corpus is of a reasonable size. Secondly, and related to the previous point, number theory had an even more marginal position in French curricula during the period. As the content of the *NAM* is known not to be limited to the curricula for mathematics in French schools, it is interesting to see how a field like number theory was treated in this type of medium. Thirdly, if number theory was mostly absent from the curricula, the borderline between number theory and algebra, which was useful for students, was often blurred. Furthermore, number-theoretical statements could be elementary and thus met the requirements of the *NAM* editors, namely to publish elementary content, « at the scope, at the level of the students ». [6] This specificity of number theory was also commented upon by one of the editors of the *NAM* in 1900, Raoul Bricard. In his review of a treatise on number theory, he recalled that « number theory frequently has statements of striking simplicity, intelligible almost without mathematical initiation, even when their demonstration has required the efforts of the most subtle invention » and that « the newcomer, bringing together the conditions of knowing how to read, having a mathematical mind and a taste for the pleasures of the abstract, can approach number theory and take part, after a few days, in the highest speculations of the human mind » [Bricard 1900, p. 477]. [7]

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[5] One result of this group research project is a database `http://nouvelles-annales-poincare.univ-lorraine.fr/` containing the number of entries published by every author and for each year, and a collective work on the *NAM* is currently being written. I will specify the methods and content of this database below.

[6] « à la portée, et à la couleur des élèves ». This quote is issued from a letter from Terquem to Eugène Catalan dated August 31, 1849, and reproduced in [Verdier 2009, p. 252]. Unless otherwise indicated, all the translations are mine.

[7] « [...] les propositions de théorie des nombres ont fréquemment des énoncés d'une simplicité frappante, intelligibles presque sans initiation mathématique, même quand leur démonstration a exigé les efforts de l'invention la plus subtile [...] Le pre-

In this paper, I focus on the multiple mathematical circulations, among authors, journals, articles, methods, and results, that emerge from the analysis of the number-theoretical content of the *NAM*, between authors, journals, articles, methods, results. I mainly rely on a corpus obtained from a systematic analysis, page by page, of the *NAM* during the period of its existence and on the recent historiography on number theory, mathematical teaching and mathematical periodicals. I will occasionally sketch out comparisons between two types of mathematical journals: French intermediate journals created from the end of the 1870s and German journals whose target audience was quite similar. [8]

2. WHAT WAS NUMBER THEORY? SOME PICTURES BETWEEN 1800 AND 1930

Before considering number theory in the *NAM*, it is important to question what the term "number theory" referred to over the period considered, particularly in France. This puts into context the results obtained for the *NAM*. Furthermore, the definitions of number theory were multiple according to the period and the mathematical communities involved. The consideration of these different definitions was the basis for the construction of the database used here.

Around 1800, two books dedicated to number theory were published, proposing two very different definitions of number theory. Adrien-Marie Legendre's *Essai sur la théorie des nombres* [Legendre 1798] was a synthesis of results conjectured and/or proved by Pierre de Fermat, Leonhard Euler or Joseph-Louis Lagrange and is focused on indeterminate analysis, identified by Legendre with number theory. In Carl Friedrich Gauss' *Disquisitiones arithmeticae* [Gauss 1801], number theory was defined as the study of the domain of integers. The book was organised according to two main arithmetical objects: congruences and forms. In the first quarter of the nineteenth century, mainly the algebraic content of Gauss' work—the algebraic solution of binomial equations $x^n = 1$—was considered; thus its influence touched mostly algebraic texts. Between 1825 and the 1860's, a research domain, entitled *Arithmetic Algebraic Analysis* in [Goldstein & Schappacher 2007a], was developed by an international network of schol-

mier venu, réunissant les conditions de savoir lire, d'avoir l'esprit mathématique et de goûter les jouissances abstraites, peut aborder la théorie des nombres et prendre part, après quelques jours, aux spéculations les plus élevées de l'esprit humain. »

8 The various journals consulted and their abbreviations are listed in the appendix: see page 64.

ars. It linked questions in number theory, algebraic equations and analysis by developing some of Gauss' results. Its objects of research were varied, including congruences, ideal numbers, series, forms, elliptic functions, etc. Most French arithmetic publications were also characterised by a connection between algebraic equations on the one hand and indeterminate equations and congruences on the other hand (without being in contradiction with arithmetic algebraic analysis). However, original disciplinary configurations also existed, for instance the approach introduced in Louis Poinsot's work [Poinsot 1845] linking number theory, algebra and geometry [Boucard 2015]. From the 1860s onwards and until the end of the century, number theory can be seen as organised in several clusters of articles, identified by analysing explicit and implicit mutual references in [Goldstein & Schappacher 2007b, 71–75]: the three main ones dealt respectively with questions studied at the beginning of the 19th century such as primitive roots or prime numbers, with number theory results according to a complex analytical approach (Dirichlet series and arithmetic functions for example) and with modular equations and arithmetic theory of forms. To a lesser extent, other networks of texts also included research on ideal theory and the theory of algebraic numbers, particularly in Göttingen. In France, several authors such as Édouard Lucas or Charles-Ange Laisant also developed an elementary number theory, embedded in combinatorics, geometrical visualisations and mathematical recreations, especially in the context of the *SMF* and the *Association française pour l'avancement des sciences* (*AFAS*) [Décaillot 2007; Goldstein 1999; Goldstein & Schappacher 2007b]. In the 1910s, French number theory was characterised by an hermitian tradition focused on theory of forms, diophantine approximation and complex analysis—a tradition that largely broke with the war—while German publications increasingly focused on algebraic number theory [Goldstein 2009].

When analysing the content of an intermediate journal, it is also useful to consider the number theoretic content that was taught to the potential readers of the *NAM*. This question is particularly complex, as it is always difficult to know the content of actual teaching from the reading of curricula or course titles. In France, secondary education had the particular property of being centralised and, in the case of mathematics, concentrated in the classes of *mathématiques élémentaires* and *mathématiques spéciales*.[9] While

[9] The classes of *mathématiques élémentaires* prepared students for entrance examinations to certain institutions such as the military *École de Saint-Cyr*; classes of *mathématiques spéciales* were intended for the candidates to the examinations of the *École polytechnique*, the *ENS*, the *École centrale* and the *École navale*.

French secondary teaching was deeply reformed between 1843 to 1925, the number theoretic content of the various curricula was very stable, reduced to a few elementary subjects: prime numbers, properties of divisibility, decimal fractions and periodic decimal fractions for the *mathématiques élémentaires* and continued fractions, resolution of indeterminate equations of the first degree and some algebraic equations such as the binomial equation for *mathématiques spéciales* [Belhoste 1995]. The situation was quite similar for the program of the *agrégation* of mathematics, with elementary notions about prime numbers and divisibility (in 1895, Fermat's and Wilson's theorems had to be known by the candidates) and themes within algebra and number theory (in 1895, binomial equations and primitive roots still were part of the subjects of the lessons). By way of comparison, the situation in Germany was different since teaching was not centralised or focused on preparing national examinations. It seems that, very occasionally, several number theory courses had been offered in various secondary teaching institutions, including themes as congruences, diophantine equations, quadratic forms, quadratic and cubic residues [Schubring 1986].

In both France and Germany, the situation in higher teaching has to be explored locally. As is known, lessons on number theory were given regularly in several German universities (at least in Berlin and Königsberg) from the 1830s [Goldstein et al. 2007]. In France, Victor-Amédée Lebesgue taught lessons on Gauss's *Disquisitiones arithmeticae* at the Science Faculty in Bordeaux in the 1860s. Number theory courses were also regularly given at the *Collège de France*: Joseph Liouville devoted several lectures in his course on definite integrals to recent research in number theory [Belhoste & Lützen 1984]. Between 1842 and 1870, more than fifteen semesters of mathematics courses in the *Collège de France* were announced as dealing with number theory by Guglielmo Libri, Charles Hermite—the lecture given by Hermite in 1849 is reported by Olry Terquem in the *NAM* the same year—and especially Liouville. [10] It is nevertheless difficult to know the effective content of this teaching and who attended these courses. At the end of the nineteenth century and the beginning of the twentieth century, some courses in number theory were also locally taught in faculties or at the *ENS*, with Jules Tannery, Eugène Cahen or Albert Châtelet in Paris [Goldstein 2009]. But number theory was not systematically taught to all students, even if geometers such as Poinsot, for example, promoted its integration into secondary education [Poinsot 1845].

10 Archives nationales, 2006/0682/030–83, Programmes de cours du *Collège de France*, from 1841 to 1870.

3. DATABASE ON THE NUMBER-THEORETICAL CONTENT IN THE *NAM*: CONSTRUCTION AND GLOBAL ANALYSIS

3.1. *Identifying Number Theory Data in the* NAM

The goal was to identify all traces of number theory in the *NAM*. The construction of the database is thus based on a systematic page by page analysis of the journal from 1842 to 1927. As suggested above, identifying what falls into « number theory » is problematic, especially in the long term. From 1842, each volume of the *NAM* ended with a table of contents organised by mathematical field. But the classification used varied over time and the way the items in it were organised also evolved.[11] To obtain a relevant and coherent corpus, I chose to make extensive use of the main mathematical catalogues available during the period: the *Catalogue of Scientific Papers* (for the nineteenth century), the *Jahrbuch über die Fortschritte der Mathematik* (from 1868) and the *Répertoire Bibliographique des Sciences Mathématiques* (1894–1912, including older works). Because the associated classifications were also unstable [Goldstein 1999, pp. 194–199] and because these works were far from being systematic in their inclusion of intermediate journals, I took into account all the entries in the *NAM* dealing with a subject classified in an arithmetic category at a given time in one of these media. Another question was that of distinguishing between number theory and elementary arithmetic: I included the items dealing with divisibility but left out those dealing with elementary arithmetic operations, such as division and root extraction. These difficulties show in an interesting way the porosity of the mathematical domains of the time, especially between algebra and number theory. The page by page analysis also allowed me to select texts where number theory was mentioned without being the main theme: these texts show interactions

11 The categories used for algebra and arithmetic in the *NAM* were as follows: from 1842 to 1849, « Arithmetic » (« Arithmetic and arithmology » in 1849), « elementary algebra », « higher algebra »; from 1850 to 1862, « algebraic analysis » and « indeterminate analysis; arithmology and arithmetic »; from 1863 to 1867, « analysis »; from 1869 to 1891, « algebra » and « arithmetic » or « arithmetic and number theory » or « number theory » according to the years; from 1892 to 1927, the classification adopted by the international congress of mathematical bibliography held in 1889 [Nabonnand & Rollet 2002] was adopted. The same object could be placed in different categories: indeterminate equations were listed in « elementary algebra » before 1850 then in « indeterminate analysis; arithmology and arithmetic » until 1862. Then they could appear either in « algebra » or « arithmetic ». The word « Arithmology » was used by Terquem, according to Ampère, and meant number theory.

mainly between number theory and algebra, geometry or recreational mathematics.

In the end, I identified 977 number-theoretic entries by 254 publishing contributors. In addition, 123 answers (solutions to problems) sent to the journal but not published were included and 43 new actors were found. I classified each entry according to its editorial form: article, reprint or translation, excerpt from correspondence, question, published or not published answer, bibliographic review, bibliographic entry without review, past examination questions and answers. [12] It should be noted that many entries were not signed. It was sometimes easy to identify the author, who was one of the editors, but some texts stayed anonymous. In addition, notes from the editors regularly supplement the articles: in this case, only the author of the article is indicated in the database. It was also difficult to know whether the authors mentioned had initiated the publication of their question in the *NAM*: this is the case, for example, of questions associated with Carl Gustav Jacob Jacobi and Ernst Eduard Kummer in 1848.

To compare number theory with the overall profile of the journal, I used the database dedicated to the *NAM* above-mentioned. For each year, it counts the number of entries per author, classified by category (articles, questions, etc.) and provides information on the authors (profession, education, place of practice, etc.). [13] Compared to my database, the search was not carried out in a completely homogeneous way. [14] The general database does not include the bibliographic lists or the authors of unpublished responses. The comparative statistics obtained are therefore only indicative and do not consider this two types of entries. In order to sketch a comparison with other mathematical journals and to identify mathematical circulations, I also used Dickson's *History of the Theory of Numbers* [Dickson 1919–1923], the online database of the *Jahrbuch*, [15] and

12 This categorisation has its limits, of course: articles are sometimes sets of answers, such as [Moret-Blanc 1881].

13 Unless otherwise stated, the biographical information of the authors given in this article is taken from the database of the *NAM* project. The additional information concerning teachers in *mathématiques spéciales* was also taken from the *Dictionnaire des professeurs de mathématiques spéciales* by Rolland Brasseur (`https://sites.google.com/site/rolandbrasseur/5---dictionnaire-des-professeurs-de-mathematiques-speciales`).

14 The analysis was done page by page from 1860 to 1895 and from the tables of contents otherwise.

15 `https://www.emis.de/MATH/JFM/JFM.html`

occasionally analysed the mathematical content of other intermediate journals, mainly from their tables of contents.

3.2. *Global Analysis: First Results*

3.2.1. *Periodisation*

Over the entire period, number theory represents about 8% of entries, but in a very heterogeneous way. Taking into account the editorial history of the *NAM* [Nabonnand & Rollet 2011] and the quantitative data on number theory entries (see mainly tables 1, 2 and 3), I constructed a three-part periodisation. This periodisation partly overlaps with the educational and editorial contexts in France and has proved relevant for the qualitative analysis of my corpus.

The first period (1842–1862) is characterised by the fundamental role played by Olry Terquem as editor of the *NAM* and by Lebesgue for his number-theoretic research. More than half of the texts were articles, original or not, and book reviews were more numerous than for the other two periods. The figures also show the timid implementation of the question-and-answer section (Q-A section)—particularly for number theory (this domain represented 2.7% of the answers against 9.7% of the questions)—whose functioning became optimal during the second period.

From 1863 to 1888, the number of arithmetical entries increased sharply, both absolutely (19 per year in average compared to 13 for the first period) and relatively (10.7% of the total number of entries compared to 6.8% for the first period). This was mainly due to the importance of the Q-A section. It was largely during this period that several answers were often mentioned for one question (almost 90% of the unpublished answers mentioned were during this period), suggesting an energetic involvement of the journal's readers. In general, it was also during this period that the journal seemed to best achieve its goals, in terms of both mathematical content and readership [Nabonnand & Rollet 2011; 2013]. We will see, however, that this must be put into perspective in the case of number theory.

From 1889 onwards, the number and proportion of number theory texts declined significantly. Several factors may explain this: fewer authors whose main research was number theory published in the *NAM*, the evolution of the editorial landscape—with the creation of journals such as the *Intermédiaire des mathématiciens* (*IM*) that contained a lot of number theory—or the 1902 secondary education reform that gave more importance to analysis and valued practical applications [Belhoste 1991; Gispert 2007]. Moreover, after 1896, the *NAM* editors favoured higher-level content to adapt the journal to the increase in university enrolments following the reforms of the late nineteenth century: number theory themes dealt with in the *NAM* texts also evolved substantially.

TABLE 1. Number of entries by type and period

Type of texts	Art	T/R	Corr	Q	A	Rev	Bib	Ex	**Total**	NPA
1842–1862	128 45,8%	27 5,8%	3 1,1%	46 16,2%	27 9,5%	40 14,1%	4 1,4%	9 3,2%	**284**	4
1863–1888	138 28,2%	5 1%	24 4,9%	131 26,8%	100 20,4%	9 1,8%	57 11,7%	24 4,9%	**489**	109
1889–1927	69 33,8%	4 2%	7 3,4%	36 17,6%	40 19,6%	11 5,4%	20 9,8%	16 7,8%	**204**	10
Total	337 34,5%	34 3,5%	34 3,5%	213 21,8%	167 17,1%	60 6,1%	81 8,3%	49 5%	**977**	123

Art = articles; T/R = translations & reprints; Corr = excerpts from correspondence; Q = questions; A = Answers; Rev = bibliographic reviews; Bib = bibliographic entry without review; Ex = past examination questions and answers, academic prizes; NPA = non published answers.

TABLE 2. Percentage of number theory in the *NAM* for articles (including reprints and translations), questions and answers

Type of texts	Articles	Questions	Answers	Total
1842–1862	8,2%	9,7%	2,7%	6,8%
1863–1888	9%	14%	10,1%	10,7%
1889–1927	4%	3,9%	4%	4%
Total	7%	9,2%	5,7%	7,1%

3.2.2. *Authorship*

Figure 1 (p. 65) shows that over the entire period of publication of the *NAM*, the journal reached its target audience: teachers and students. Compared to the overall content, number theory had a much higher proportion of teachers among authors, as Table 3 shows for the first period. The domination of teachers persisted over the following two periods, although a little less strongly. The contrast is particularly interesting on the answers given to the questions, especially in the second period: 58% of the authors of the questions were teachers while 15% were students (against 34.5 and 35% respectively for the whole journal). From this perspective, number theory seemed to involve teachers to a much greater extent and exchanges around questions and answers were mostly between peers. We will see in section 5.3 that during the second period, the Q-A section was indeed invested by a few geometers to study indeterminate equations or properties of Bernoulli numbers for example. The other categories represented are mainly engineers and military, which are much lower (between 2 and 10% for engineers and 5 and 11% for military), and quite similar to the global situation.

TABLE 3. Participation of teachers and students in the *NAM* for the first period (1842–1862). For each case, the first percentage corresponds to teachers and the second to students.

	Authors	Articles	Questions	Answers
Number Theory	59% / 20%	77%/8%	54% / 0%	39% / 35%
Total	37% / 37%	41.5%/17%	52% / 5.5%	27.5% / 45.5%

Among the 254 contributors, 15 authors published more than ten times in the *NAM* and seven authors less than ten times but at least four articles (see Table 4). Only four of them were not teachers at some point of their career. Beyond these quantitative data, their status in relation to number theory in *NAM* was very different. 5 were editors of the *NAM*: Terquem, Camille Gerono, Laisant, Eugène Prouhet and Bricard. Their activity in number theory in the *NAM* was not significantly above average. However, their role as editors may have been important, as in the case of Terquem. Others, such as Claude Moret-Blanc or Georges Fontené, were very regular authors of the *NAM* and published in number theory as well as in other mathematical fields. Finally, the important arithmetic production of some authors in the *NAM*—more than 25% of their published entries—reflected the general orientation of their work: this was the case, for example, of Savino Realis, Lebesgue and Lucas. [16]

3.2.3. *Number-theoretical texts in the* NAM

Number theory in the *NAM* was multiple: subjects included in the curriculum, such as elementary questions of divisibility, periodic decimal fractions or indeterminate analysis of the first degree; higher arithmetic, such as the theory of quadratic residues, algebraic integers or geometry of numbers; and mathematical recreations for example. If indeterminate analysis

[16] The case of Eugène Lionnet is unusual: of the 42 published entries, 28 were questions of which at least ten from his *Traité d'algèbre élémentaire* (1868).

constituted the dominant topic studied during the whole period, it represented nearly half of the published entries during the second period, especially because of the increasing number of questions and answers. On the contrary, subjects as congruences, cyclotomy or algebraic number theory, have a more important place in the journal during the first and third periods.

Different types of articles can be identified. Some of them were explicitly intended for the preparation of the examinations, recalling a traditional process or introducing an interesting method, i.e. original, simple or efficient in terms of calculation. Others proposed generalisations of topics on the curricula. [17] The last ones proposed a popularised version (to some extent) of statements or theories to complement the mathematical education of readers. As we will see from examples, the notion of elementary had varying meanings in the *NAM* for number theory: some texts may have been elementary in the sense that they dealt with concepts taught in classes of *mathématiques élémentaires* and *spéciales* and were therefore supposed to be known to students; others were elementary because they could be understood by a student of *mathématiques spéciales* without being part of the *curriculum*; some were elementary because they were based on (relatively) elementary results of number theory but required arithmetic knowledge outside the text and curriculum.

Translations and reprints of texts already published elsewhere were specific to the *NAM*. This editorial practice, often at the initiative of the editors, was part of a more general movement of mathematical translations in the nineteenth century [Chatzis et al. 2017]. In the case of number theory in the *NAM*, I identified 37 reprints and translations, listed in the Table 8.3 (p. 67). [18] A large majority of these texts (27 of 36) were published during the first period, between 1842 and 1862. This represents about 20% of the number theory articles published then and the translations from German correspond to about 9% of the articles, which corresponds to the general case for the *NAM* [Chatzis et al. 2017, p. 16]. After 1863, very few texts

17 This overlaps with the conclusions for a set of European intermediate journals given in [Ehrhardt 20??].

18 Here, I consider the reprints and translations of articles or answers explicitly mentioned—by the mention « After » in the title or by the indication of the name of a translator—and resulting from a previous publication. By translations, I mean complete, but also partial translations, reformulations or even summaries. For example, I did not consider as « translation » the articles by the Japanese mathematician Tsuruichi Hayashi that are original but translated from English or one of Charles Hermite's solutions published in the *Quarterly Journal of Pure and Applied Mathematics* in 1857 that Terquem summarised following another solution.

were announced as taken from other journals or treatises. The temporalities involved were also very different between the first period—on average about fifteen years between the original publication and publication in the *NAM*—and the next two—a maximum of one year. This difference reflects different goals from the editors or the translators: to train the readership in theories and methods already known but not in the curricula or to keep the readership informed of a certain mathematical topicality, in order to foster exchanges within the *NAM* or to enrich its mathematical culture.

As for the articles, questions concerned a wide range of subjects. They were considered as an important part for the training of students since one of the important skills expected for competitions was problem-solving. [19] More generally, exercise solution was seen as the best possible training for a mathematics student [Laisant & Antomari 1896]. They could also allow a form of competition between students, or even between certain institutions [Ehrhardt 20??]: for example, three students explicitly identified as being in Charles Briot's class at the *Lycée Saint-Louis* answered arithmetical questions between 1862 and 1865. [20] The questions about indeterminate equations or divisibility for example could be interpreted as training for students as they were on topics that were part of their formation or else their solution implied the use of algebraic tools. Questions could be directly linked to the content of the curricula or not. Questions could also be linked to previously published articles. For example, Fontené wrote an article on algebraic numbers in 1903, and one of his questions published in 1910 concerned the same theme. Some questions were also unsolved problems, such as Catalan's conjecture [21] in 1842. Regarding the level of questions asked in the *NAM*, Lebesgue wrote to Jules Houël in 1869 to indicate his disagreement with Justin Bourget, then editor of the *NAM*, who wished to see in his journal only questions in accordance with curricula of the class of *mathématiques spéciales*:

> « I told Mr. Gerono who communicated my letter to Mr. Bourget that many of the questions were not intelligible to the candidates and even to some professors. I don't think Mr. B. should stick to Math. Special, many questions are put in for young professors who have time to work. » [22]

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19 The practice of problem-solving was far from being specific to France: see [Despeaux 2014].

20 At least three other students of the same high school also answered arithmetical questions in the *NAM* and were probably in Briot's class.

21 Two consecutive integers, except 8 and 9, cannot be exact powers.

22 « J'ai dit à Mr Gerono qui a communiqué ma lettre à Mr Bourget que bien des questions n'étaient pas intelligibles par les candidats et même pour certains professeurs. Je ne crois pas que Mr B. doive s'en tenir aux Maths. Spéciales, beaucoup

At the same time, Lebesgue regularly asked questions in the *NAM* about cubic or biquadratic indeterminate equations, a priori more intended for his fellow arithmeticians than for high school students. A significant proportion of questions were not original, but how much is difficult to quantify. This was the case, for example, of Lionnet's ten questions published in 1868 explicitly from his *Traité d'algèbre élémentaire* published the same year. Some questions could be taken from foreign publications and translated: for example, a question on primitive roots originally published in the Russian treatise of Pafnuti Čebyšev on congruences in 1849 was translated for the *NAM* in 1856. Questions or answers from other journals were also reproduced, as with the *Educational Times* (*ET*), from which several questions, well adapted to the readership of the *NAM*, were reproduced in the *NAM*, particularly in the 1870s. Another type of question, particularly designed for students, was derived from the examinations, reproduced in the *NAM* often along with a suggested correction. In the case of number theory, however, they were few in number, mainly from the entrance examinations to the *École de Saint-Cyr* and the *École navale* on the curriculum of *mathématiques élémentaires*, then to the *agrégation* and the *concours général* from the 1870s onwards. These arithmetical questions could probably destabilise the candidates: in 1858, Gerono indicated that at least two questions asked for admission to the *École navale* made it possible to test the candidates' « spontaneity » (I, 17, 353, 396).

The bibliographical section of the *NAM* informed readers of recent publications, including research articles and textbooks. Between 1870 and 1890, it consisted mainly of bibliographic lists without any comments, summarizing in particular the content of several research journals such as the *Mathematische Annalen*, the *Proceedings of the London Mathematical Society* or the *American Journal of Mathematics*. Two main types of books were reviewed in relation to number theory: on the one hand, during the first period, those that analysed the content of arithmetic or algebra treatises and commented on the presence or absence of number theory elements; and on the other hand, during the third period, those that concerned recent number theory books. This echoed the scarcity of French number theory books in the second half of the nineteenth century.

de questions sont mises pour les jeunes professeurs qui ont le temps de travailler. » Archives de l'Académie des sciences de Paris, Dossier V.-A. Lebesgue, Letter from Lebesgue to Houël, February 3, 1869. I thank Norbert Verdier for sending me the transcript of this letter.

4. PROMOTION AND DIFFUSION OF NUMBER THEORY FOR STUDENTS AND TEACHERS IN THE *NAM*, 1842–1862: THE ERA OF TERQUEM AND LEBESGUE

Among the seven authors publishing more than five entries in the *NAM* between 1842 and 1862, Terquem and Lebesgue largely dominated arithmetic publications. They both were previously regular authors and translators for the *Journal de mathématiques pures et appliquées* (*JMPA*). However, their profiles as authors in the *NAM* were very different as suggested by the distribution of their types of intervention. Editor of the *NAM*,[23] Terquem was a frequent contributor to his journal, especially *via* reviews, translations and reprints, as well as notes and additions of references to texts by other authors. While not specific to number theory, his multiple interventions provide information on the objectives of integrating number theory into the *NAM*. His references and notes allowed readers to make links between arithmetic publications, within the *NAM* but also with external publications. He also introduced a new symbol $\dot{p}$ to designate any multiple of an integer p which replaced the notation $\equiv$ of Gauss congruences, for scientific and typographical reasons [Boucard 2015]. This symbol was used in a majority of arithmetical articles until Terquem's death, and still on an occasional basis during the following period. For all these reasons, the number theory in the *NAM* can then be considered to be Terquem-coloured.

Lebesgue was one of the few authors of the *NAM* who specialised in number theory. Number theory dominated his whole research production, published in various journals as the *CRAS* or the *JMPA* [Abria & Houël 1876]. In the 1830s and 1840s, the themes he addressed were original compared to French arithmetic publications. Lebesgue was recognised as an arithmetician by his peers and became an associate correspondent of the Academy of Sciences thanks to his number theoretical work [Boucard 2015, pp. 530–531]. His role for number theory in *NAM* went beyond his own publications. He regularly exchanged with Terquem [Verdier 2009, pp. 206–207], who described him as a distinguished « arithmologist » [Finck & Deladéréere 1844, p. 41] and he answered Gerono's arithmetical questions very regularly in their correspondence. Thus, in a letter dated February 1, 1869, Gerono confided to him: « I think we would do well to

[23] It should be noted that Gerono, as the second editor of the *NAM*, was also involved in the dissemination of number theory in the journal, to a lesser extent. His interventions were more numerous at the end of the period, while those of Terquem had almost disappeared.

consult you about the articles we publish on numbers; you know more than we do on the subject ». [24]

4.1. *The Interest of Number Theory for Forming the Mind and Simplifying Theories*

In the 1840s, Terquem and some others pointed out the value of introducing number theory into education. This field had been described on several occasions as having a specific role in the training of students, particularly in the context of comments on books. In Fournier's review of *Éléments d'arithmétique et d'algèbre*, Terquem indicated, for example, that the most appropriate field for exercising intellectual faculties was number theory [Terquem 1844, p. 140]. Specific to intelligence training and « true dynamometer of genius » [Terquem 1847, p. 133], number theory could also be used to select the best candidates:

> Euler believes that number theory particularly is more able even than the geometry to give rectitude to the judgment and intensity to the meditative faculty. If we introduced the main propositions of this theory into classical education, examiners would acquire an excellent criterion for classifying intelligences. [Terquem 1846, p. 78] [25]

Lebesgue also stressed the importance of including higher algebra and number theory in university teaching, even if this field might seem useless at first sight: « the curiosities of science quite often one day become necessities » [Lebesgue 1852, p. 420]. [26]

This critical position towards educational programs was a recurring one for Terquem. He regularly protested against excessive utilitarianism and the growing importance of industrial applications in education. He became particularly virulent in 1850 when new programs for the *École polytechnique* were established, more practice-oriented [Verdier 2009, p. 169]. Terquem explained in particular the absence of number theory in teaching by the abstract and non-applied nature of this field, in a commentary on Kummer's recent arithmetical memoirs:

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24 Archives de la Bibliothèque de l'Institut de France, MS 2031 (Bertrand), Correspondance de Gerono à Lebesgue.

25 « Euler croit que la théorie des nombres en particulier est plus propre encore que la géométrie à donner de la rectitude au jugement et de l'intensité à la faculté méditative. Si l'on introduisait les principales propositions de cette théorie dans l'enseignement classique, les examinateurs acquerraient un excellent criterium pour classer les intelligences. »

26 « [...] les curiosités de la science en deviennent assez souvent un jour des nécessités. »

> Note. The theory of numbers is very little practised and often even scorned in France and for good reason. This theory doesn't help to turn wheels, to open floodgates, it condenses neither gas nor vapor, and, what is worse, is useless for examinations; therefore, calculating minds, who are everywhere the majority, are entitled to ask why study a theory that is profitless. My limited intelligence has no reply to such questions. [Terquem 1850, pp. 391–392] [27]

In addition to these general considerations on number theory, some arithmetic results were seen as important for students: Fermat's and Wilson's theorems [Catalan 1842; Terquem 1842; 1844] or residue theory [Prouhet 1846] and congruences [Midy 1845] for example. On the one hand, this would have simplified and generalised the presentation of concepts in the program, such as divisibility or periodic fractions. Terquem repeatedly insisted on the interest of residue theory in approaching periodic decimal fractions [Terquem 1848, p. 114] or divisibility more generally, quickly and simply [Terquem 1848, p. 112]. These statements were therefore not used to obtain new results but to provide a more appropriate presentation of concepts taught in class. Catalan also used this type of argument to justify his article on periodic fractions, based on Fermat's theorem [Catalan 1842, p. 457]. In this article, Catalan also indicated that he was partially reproducing a work on periodic fractions published by Étienne Midy in 1835 that was very difficult to access [Midy 1835]. He also published several articles in the *NAM* in the 1840s. In the one on the indeterminate analysis of the first degree, Midy stressed the importance of solving the equation $ax \pm by = \pm c$ for « young mathematicians », « by the many applications that can be made of it and the calculation tricks it is likely to use » [Midy 1845, 146]. [28] This justified the different methods set out in the treatises on algebra on this subject. The method proposed here by Midy is seen as « easier and quicker » to investigate [Midy 1845, p. 147] [29] and will make it possible to « familiarise students with principles whose application, still not widespread, can be very useful, and whose judicious use is very specific to exercising their sagacity » [Midy 1845,

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27 « Note. La théorie des nombres est peu cultivée, et communément même dédaignée en France, et pour de bonnes raisons. Cette théorie ne fait pas tourner des roues, ne fait pas ouvrir des vannes, ne fait condenser ni gaz ni vapeur, et, ce qui est encore pis, ne sert pas aux examens ; dès lors les esprits calculateurs, partout en majorité, sont en droit de demander à quoi bon étudier une théorie qui ne rapporte rien. Mon intelligence bornée ne me fournit aucune réponse à de semblables questions. » Quote from [Alfonsi 2012].

28 « [...] par les nombreuses applications qu'on peut en faire et les artifices de calcul dont elle est susceptible »

29 « [...] plus facile et plus prompte »

p. 147].[30] Terquem noted that he published this article in the *NAM* to « propagate » the method of the « congruences of M. Gauss » [Midy 1845, p. 147]. The student readership of the journal was thus explicitly targeted in these arithmetic papers.

On the other hand, Prouhet, in a paper on residue theory, justified his work by its usefulness to « young readers », not only to present traditionally isolated theorems « in a uniform manner » but also to « facilitate [...] access to number theory, a difficult and as yet uncultivated part of mathematics, on which the future of science today seems to rest. » [Prouhet 1846, p. 178].[31] With Prouhet, number theory was the main focus of the paper, not a method for setting out curriculum statements. His goal was to present in a coherent way a set of arithmetical results, already known but scattered in older and outdated treatises, difficult for students to access, such as Legendre's *Théorie des nombres*. This echoed other comments published in the *NAM*, suggesting the difficulty of introducing number theory to students. In his treatises on arithmetic and algebra, Terquem regularly deplored the absence of number theory results and the use of inadequate methods or, on the contrary, pointed out the presence of a particular statement. Terquem was for example laudatory about Joseph Bertrand's *Traité d'arithmétique* [Bertrand 1849], because « Arithmology finally enters the fundamentals. This is progress. » [Terquem 1849a, p. 315][32] The absence of a recent French treatise on number theory was regretted several times in the pages of the *NAM*. The material and financial difficulties inherent in such an editorial project were highlighted by Terquem when he announced the project of a « methodical and complete treatise on indeterminate analysis » by the « deep arithmologist of Bordeaux » Lebesgue [Lebesgue 1847, p. 431]. In the following years, regular references to the publication of Lebesgue's books on number theory and associated financial obstacles were published in the *NAM*.[33]

30 « Elle aura d'ailleurs l'avantage de familiariser les élèves avec des principes dont l'application encore peu répandue, peut être fort utile, et dont l'emploi judicieux est très propre à exercer leur sagacité. »

31 Les théorèmes que nous démontrerons ne sont pas nouveaux ; à quelques développements près, ils se trouvent dans l'ouvrage de Legendre, mais séparés et déduits de théories différentes. Nous avons cru utile de les réunir et de les déduire les uns des autres d'une manière uniforme. Ce travail aura en outre l'avantage de faciliter aux jeunes lecteurs de ce journal, l'accès de la théorie des nombres, partie difficile et encore peu cultivée des mathématiques, et sur laquelle paraît aujourd'hui reposer l'avenir de la science.

32 « L'arithmologie pénètre enfin dans les elements. C'est un progrès. »

33 Lebesgue did indeed encounter great difficulties in order to be able to publish his books [Verdier 2009, pp. 123–124] and in particular called on the academicians

The importance of number theory for the readership of *NAM* was therefore supported by several arguments. First, given its limited place in secondary school curricula, this field also occupied a marginal place in most algebra and arithmetic textbooks. The number theory treatises of the early part of the century were more difficult to access and did not reflect more recent work. Also, several authors argued in favour of a more substantial introduction of number theory in these programs for several reasons: it made it possible to simplify and generalise several theories; it was particularly suitable for training minds and therefore for selecting the best candidates. Finally, number theory was fundamental to mathematics and therefore useful for the mathematical culture of readers. As a little cultivated and abstract field, number theory also served Terquem's critics against the practical and industrial orientations sought in recent *Polytechnique* programs. For all these reasons, several authors published in the *NAM* texts allowing readers to easily access—from an intellectual and material point of view—elementary statements of number theory. It was during this first period that we found the largest number of translated texts related to number theory « to the students' colour ».

4.2. *Mathematics For Examinations and Beyond: the Case of Number Theory*

Several groups of texts dealt directly with curricula: these were, for *mathématiques élémentaires*, criteria of divisibility, determination of prime numbers with the Sieve of Eratosthenes and variants or periodic decimal fractions; continued fractions and indeterminate first-degree analysis for *mathématiques spéciales*. These texts are mostly published by high school teachers, but also by a few students. As mentioned above, they sometimes aim to simplify already known results—such as Catalan on periodic fractions—and sometimes propose original methods. The solution of the indeterminate equation $ax + by = c$ was the subject of several articles, based on different methods: use of the largest common divisor [Chevillard 1843; Dieu 1850], use of tables [Catalan 1844] and residues [Midy 1845]. In each case, the methods were illustrated with non-trivial numerical examples. A question, published in 1858 and attributed to Hermite, focused on the equa-

to plead his case to the Minister of Public Instruction. Lebesgue and Gerono also exchanged on this point in their correspondence (Bibliothèque de l'Institut de France, MS 2031, Correspondance de Gerono à Lebesgue) and Lebesgue's career file contains several documents testifying to these complications, such as a letter by Academicians adressed to the minister to advocate Lebesgue's case (Archives nationales, F/17 /3175, Dossier Lebesgue).

tion of the first degree but from a different perspective: the question was to determine the number of whole and positive solutions of the equation $ax+by = n$, with n given. A student at *Lycée Saint-Louis* used a method based on algebraic manipulations from Bertrand's *Traité d'algèbre élémentaire* [Bertrand 1851, pp. 206–207] to propose an answer, published the same year. This was supplemented by Terquem with a method, due to Hermite, based on manipulations of formulae on integer parts, published in 1857 in the *Quarterly Journal of Pure and Applied Mathematics*. It was therefore likely that Terquem had inserted this question in his journal in order to be able to publish an elementary solution that was different from those of classical algebra textbooks.

The previous notions were also the subject of articles proposing developments going beyond curricula. This was the case for about ten articles on the number of steps of the Euclidean algorithm, published in the *NAM* between 1842 and 1851. These texts resonated directly with notes published in the *CRAS* and *JMPA* at the same time [Verdier 2009, pp. 251–252]. Similarly, the distribution of prime numbers was addressed through texts on the theorem of arithmetic progression.[34] Honoré Adolphe Desboves recalled Legendre's incomplete demonstration and quoted Johann Peter Gustav Lejeune Dirichlet's memoir containing a complete proof based on infinite series [Dirichlet 1839]. Desboves also proposed a generalisation of the Čebyšev theorem,[35] which he considered to be a « useful exercise to recommend to young readers of the *Nouvelles Annales* » [Desboves 1855, p. 293]. On the same theme, Lebesgue also relied on recent memoirs by Dirichlet and Serret which contained elementary proofs of particular cases of the theorem of arithmetic progression [Lebesgue 1856]. Desboves and Lebesgue, while knowing and quoting Dirichlet's work based on an analytical approach, proposed content on the same subject—the distribution of prime numbers—adapted to the *NAM* readership.

The links between number theory and other fields were also regularly highlighted, particularly by Terquem. In an article by A. Vachette on polygons, Terquem insisted, for example, on the links between geometry and number theory *via* the « géométrie de situation », a theme he revisited in 1849 when he summarised Poinsot's work on polygons and polyhedra [Bou-

34 This theorem, already stated by Legendre in the 18th century, reads as follows: For any non-zero integer n and any integer m relatively prime to n, there is an infinite number of primes of the form $kn + m$.

35 This theorem, conjectured by Bertrand in 1845 and proved by Čebyšev in 1850, also concerns the distribution of prime numbers: for any integer n greater than 2, there is a prime number p strictly between n and its double.

card 2017, pp. 88–90]. Terquem also drew attention to the links between algebra and number theory in several memoirs on the algebraic theory of equations *via* binomial equations (and in particular, cyclotomy) and the search for whole roots for algebraic equations of degree greater than 2, in the program of *mathématiques spéciales*. This led to the introduction of objects from number theory—primitive roots of prime numbers, congruences, Fermat's theorem—and results on complex integers $a+b\sqrt{-1}$. Terquem noted for example the link with congruence theory in a paper published by Realis, then a student in Paris [Catalan 1886], on the elementary algebraic solution of the equation $x^p - 1 = 0$ (p prime). Still on the theme of the cyclotomic equation, Lebesgue relied on results contained in his earlier memoirs and Gauss' *Disquisitiones arithmeticae* and explicitly positioned his paper between algebra and number theory with an application to congruences and quadratic residues [Lebesgue 1852].

Other memoirs dealt directly with number theory for its own sake and outside the curricula. Again, Terquem acted as a facilitator and regularly added internal references to *NAM* and other publications. This made it possible to ensure consistency between the various texts and located them in relation to past or current publications outside the *NAM*. As previously reported, recent elementary sources on number theory were missing and several original texts explicitly aimed to fill this gap for residue theory (Prouhet in 1846, Lebesgue in 1852), Wilson's theorem (Lionnet in 1842, Terquem in 1843, Prouhet in 1845), congruences (Terquem in 1844, Lebesgue and Prouhet in 1850, Lebesgue in 1852). The whole constitutes an original reception of the elementary number theory publications of the first half of the 19th century for the readers of the *NAM*, i. e. by proposing an elementary content in the sense of Terquem [Boucard 2015; Boucard & Verdier 2015]. Special mathematical tools could also be used to demonstrate number theory results: Hermite thus proposed a demonstration of the theorem of the two squares [36] based on continued fractions in 1853, already published in the *JMPA* in 1848. Arithmetical conjectures were also published in the *NAM*. The Catalan conjecture has already been mentioned. As part of a set of texts on the decomposition of numbers into a sum of similar powers, Terquem recalled in 1856 Waring's conjecture on the possibility of decomposing any number into a sum of 19 biquadratics (possibly including zero) and the recent numerical verification work commissioned by Jacobi and carried out

36 A prime number of the form $4n+1$ is always the sum of two squares. This theorem, already demonstrated by Euler in the 18th century, was the subject of numerous demonstrations in the *NAM*.

by Carl Anton Bretschneider up to 4100, published in the *Journal für die reine und angewandte Mathematik* (*JRAM*) in 1853.

4.3. *Transcribing Mathematics for Teachers and Students*

The analysis of the 27 translations and reprints published in the *NAM* under the direction of Terquem (and Gerono) reveals three major characteristics.

First of all, Terquem played a central role in this respect: in most cases where the name of the translator was not mentioned, Terquem seemed to be at the origin of the translation, given his German skills. Similarly, he was certainly at the origin of the various partial reprints. Many of these texts were part of the strategy developed by Terquem in his journal, to feed ongoing arithmetic exchanges or to present number theory results to readers.

Second, the number of years between the translation in the *NAM* and the initial publication was extremely variable. This echoed Terquem's desire to make classical arithmetic results and demonstrations by Euler, Gauss, Dirichlet or Jacobi, for example, accessible to the readers of his journal. Articles translated or reproduced more quickly most often made it possible to complete other publications of the *NAM* and to offer the reader recent arithmetic results adapted to the readership. This was the case, for example, with Leopold Kronecker's paper on the irreducibility of the cyclotomic equation [Terquem 1849c]: Terquem summarised the proof of a well known result and demonstrated earlier by Gauss and Legendre among others: the interest of Kronecker's version was that it « [seemed] simpler » [Terquem 1849c, p. 421]. Terquem also referred to this translation in a note on a article by Prouhet on the same subject the following year.

Finally, we find the different forms of translation identified in [Chatzis et al. 2017, pp. 21–22]: the notion of « transcription » which is introduced and understood in the sense of a textual arrangement for adaptation to a specific readership is particularly well adapted for this period. There were four literal translations of a complete text and only one was inserted in the *NAM* without any comment, the paper of Gotthold Eisenstein on algebraic equations translated by Lebesgue. Émile Coupy introduced his translation of Euler's paper on the Königsberg bridge problem with a note, referring in particular to a paper by Poinsot and its summary published in the *NAM* two years earlier. The others were all embellished with notes added by Terquem. Edmond Laguerre's translation of Jacobi's 1837 memoir published in 1856 [Jacobi 1856] illustrated this particularly well: the initial text was enriched with three successive series of notes, by Jacobi, Laguerre and finally

Terquem, which informed the reader about recent and particularly French publications related to the memoir [Boucard & Verdier 2015, pp. 68–69]. Finally, many of these translations or reprints were in fact more or less important reformulations, selections of extracts or even summaries produced by Terquem. This was the case, for example, of the text on polygons and polyhedra written by Terquem « according to Mr. Poinsot », which did not quote and did not even reproduce the organisation of the original text.

The objects of these translations could be relatively high level number theory papers: for example, Jacobi's aforementioned memoir contained fundamental results on Gauss and Jacobi sums, as well as applications to the law of cubic reciprocity and quadratic forms. They could also complement exchanges on regularly discussed topics such as continued fractions. Following sometimes heated exchanges on continued fractions between Guilmin and Catalan in 1845 and 1846, Terquem included translations of three articles by Thomas Clausen, Eisenstein and Carl Wilhelm Borchardt on the same subject. In his summary of Clausen's 1846 paper, Terquem draws conclusions on the relationship between properties of continuous fractions, greatest common divisors, recurrent series and indeterminate analysis highlighted by an Euler algorithm. On this subject, he referred to one of his own articles published in the *JMPA* [Terquem 1839] and planned to publish an amended version for the *NAM*, a project that does not appear to have been pursued. Conversely, when he summarised Borchardt's 1854 note, Terquem explicitly called on Liouville to publish in his *Journal*. We thus find the well-known exchanges between the two editors about their respective journals [Verdier 2009, pp. 266–268].

4.4. *Two Examples of Arithmetical Circulations in the* NAM *and Elsewhere: Complex Numbers in Number Theory and Products of Consecutive Integers*

4.4.1. *Complex Numbers and Number Theory*

Three themes dealt with in the *NAM* were the occasion for an introduction to the questions on the use of certain classes of complex numbers in number theory and on the arithmetic properties associated with them. As early as 1843, in a memoir on the roots of equations, Terquem introduced Gauss' notion of « complex integer root » *via* a final note [Thibault 1843, 527]. In 1844 he merged two memoirs on the complex roots of equations into one article which he introduced with a long note on the notion of *complex root* with references to Gauss' and Dirichlet's number theoretic papers. The article then began with five paragraphs devoted to the arithmetic of complex integers $a + b\sqrt{-1}$ before focusing on algebraic equations. In the same volume, Lebesgue published a short note on the same topic [Lebesgue 1844] in which he announced that he would develop Dirichlet's results on complex integers to determine the complex integer roots of algebraic equations in a manner similar to the real roots. Thus, Lebesgue planned to select and reformulate an extract from a number theory paper by Dirichlet of which « the main proposals are completely outside the scope of the *Annales* » [Lebesgue 1844, p. 146] in order to transpose a valid method for real roots to imaginary roots. Moreover, from these memoirs on the algebraic theory of equations, Terquem and Lebesgue thus introduced the readers of *NAM* to the arithmetic of complex integers. Terquem returned to these arithmetical objects in his three-part number theory article published the same year [Terquem 1844, p. 340].

Complex integers were also mobilised in the early 1850s, but this time as part of research on indeterminate analysis: Lebesgue, in 1850, demonstrated a particular case of the Catalan conjecture, with the impossibility of the equation $x^m = y^2 + 1$ and used Gaussian integers to do so by returning the reader to Terquem [1844]. Similarly, in 1853 and 1854, Angelo Genocchi published two notes on the number of possible decompositions of a number into two squares using complex integers, following Gauss [Genocchi 1853; 1854]. In the memoir translated in the *NAM* in 1856, Jacobi also worked with the sums of Gauss and Jacobi, which are linear expressions in the powers of the roots of the binomial equation. He presented fundamental results on Gauss and Jacobi sums, and applications to the law of cubic reciprocity considering in particular numbers of the form $x + y\sqrt{-3}$ and quadratic forms.

Finally, Terquem recounted several times the mathematical topicality of Fermat's last theorem between 1847 and 1850 [Verdier 2009, pp. 292–293] and recalled that the current leads were based on the question of the uniqueness of the composition in prime factors of certain complex numbers [Terquem 1849b, p. 364]. Similarly, Lebesgue reviewed the methods of solving the Fermat equation, distinguishing between the particular cases that could sometimes be solved using the infinite descent method or a method he called « non-congruence », and the general case, which required a study of the imaginary « expressions, formed with roots of the unit and called complex numbers » [Lebesgue 1847, pp. 430–431]. In 1850, Terquem returned to Kummer's work on these same complex numbers, whose arithmetic depends on divisibility properties involving Bernoulli numbers.

These few texts echoed to some extent the arithmetic research carried out since the 1830s by Jacobi, Dirichlet and Kummer in particular, involving reflections on new classes of numbers, complex integers and cyclotomic integers. The readership of *NAM* was thus familiarised with current issues in number theory, based on memoirs published in the *NAM* and references to writings published in other periodicals.

4.4.2. *A Question and its Transformations*

The second example focused on an arithmetic question asked in the 1857 naval school entrance examination: « The product of four consecutive integers cannot be a square » (I, 16, 393–394). This example is interesting because it shows how a question and its variants could generate a wide variety of answers, from the point of view of actors and methods. A series of questions, answers and articles were published on the subject between 1857 and 1862. It contains variants of the initial question related to the number of consecutive integers considered: the *polytechnicien* and military man Henri Faure had proposed a few months earlier question 386—« The product of three consecutive integers cannot be neither a square nor twice a square » (I, 16, 183)—; an anonymous question concerned the product of five or six consecutive integers (Q416, I, 17, 31); Gerono dealt with the case of seven integers in 1860 and Pierre-Adolphe Guibert (then examiner at the Naval School) the cases of eight to eleven integers in 1862. These variants would also concern the type of sequence involved—statements by Victor Berton (Department of the Navy) in 1859 and Guibert (then an examiner at the Naval Academy) in 1862 concerned products of consecutive terms in an arithmetic sequence—or the form of the product—Lebesgue showed in 1860 that the product of five consecutive numbers could not be a cube. Finally, Terquem and Émile Mathieu, then a student at the Sorbonne, proposed the general statement on the impossibility that a product of consecutive integers could be a perfect power, the first in a note of Faure's statement and the second as a question in 1858 (Q441, I, 17, 187).

The methods used to prove these different statements were also diverse. Three answers were published in 1857 and 1858 on the cases of three and four consecutive numbers: Adolphine D. proposed a solution for the case of three integers, Gerono and P. A. G. (perhaps Guibert, who could be at the origin of the question asked in the naval school entrance exam) for the case of four integers. All three solutions were based on algebraic manipulations and elementary considerations of divisibility. In 1859, Berton used the same type of algebraic identities to show that the product of four integers in arithmetic progression was not a fourth power. Mathieu answered his own question the same year, using the Čebyšev theorem mentioned earlier, and referred to a more general note by Liouville published in the *JMPA* [Liouville

1857]. Two years later, Gerono and Lebesgue proposed solutions based on the study of an associated indeterminate equation and on the consideration of the properties of decomposition into prime factors of perfect power respectively. The various authors did not position themselves in relation to the other methods proposed, but it is likely that Lebesgue and Gerono wanted to propose an elementary method for dealing with particular cases, more likely to be asked of candidates, and not based on the Čebyšev theorem. This question was again the subject of two texts some ten years later in the *NAM*: Désiré André published a thesis in 1871 on the impossibility of the equation $x(x+1)(x+2)\cdots(x+n-1) = y^n$ $(n > 0)$ in strictly positive integer numbers [André 1871] based on considerations of divisibility. Moreau reacted in 1872 by recalling the solution of the problem proposed by Mathieu in 1858 based on the theorem of Čebyšev.

During this first period, the *NAM* offered readers various kinds of number theory content, ranging from simple methods for solving classic examination questions to original memoirs on topics discussed simultaneously at the Academy or in other journals such as *JMPA*. Terquem regularly published summaries and notes that allowed readers to get to know a certain topicality of number theory, even if it was not always elementary: this is the case, for example, of Kummer's research on Fermat's last theorem. Nevertheless, entire parts of number theory remain invisible through the *NAM*: the analytical number theory or the dozens of articles published by Liouville in his *Journal* from 1856 on the representation of integer numbers by particular quadratic forms [Verdier 2009, pp. 322–338]. As a very partial comparison, let us turn to another intermediate journal created in 1841 by Johann August Grünert, the *Archiv der Mathematik und Physik*, whose target readership was a priori similar to that of the *NAM*.[37] The functioning of this journal was quite similar to that of the *NAM*. If I did not identify any appropriation of the content of the *JRAM* for number theory as could be the case of the *NAM* with the *JMPA*,[38] contents of the *JMPA* and *NAM* were very regularly included: Poinsot's work on polygons summarised by Terquem or arithmetic questions published on the *NAM* were for example the source of articles in the *Archiv* by J. Dienger in 1849 and 1851, respectively. On the contrary, I did not find any references to the *Archiv* in the number theoretic entries published in the *NAM*. Like

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37 The *Zeitschrift für Mathematik und Physik*, created in 1856 by Oskar Schlömilch, was also an intermediate journal, but containing only a very small number of arithmetical articles during the early years.

38 This could be explained by a strong enmity between Grünert and Crelle [Verdier 2009].

Terquem, Grünert seemed to play an important role in disseminating external publications to his readers: for example, in 1846, he summarised several arithmetical memoirs by Louis-Augustin Cauchy and Poinsot. Themes similar to *NAM* were addressed in a recurrent manner: divisibility, periodic decimal fractions, Fermat and Wilson theorems or indeterminate equations. Like Lebesgue for the *NAM*, some authors stood out for their arithmetic production: this was the case of Friedrich Arndt who published about ten memoirs on the theory of quadratic and cubic forms, a theme virtually absent in the *NAM*. These few elements suggest that there would be great interest in conducting a more exhaustive comparative study between these two mathematical journals.

For the *NAM*, this period ended with Terquem's death and this was the end of an original editorial practice for number theory in the journal. Lebesgue also published a series of three articles in arithmology to echo Terquem's words in praise of his colleague:

> My excellent and very unfortunate friend Mr. Terquem liked the words *arithmologist, arithmology*: that's what decided me to give this article and others that will follow it, the title it bears and which would have been clearer in these terms: *Elementary research on integers*. [Lebesgue 1862, pp. 219] [39]

5. *NAM* BETWEEN 1863 AND 1888: A JOURNAL FOR INDETERMINATE EQUATIONS

5.1. *A Transformed Educational, Institutional and Editorial Context*

In the 1860s, science was considered increasingly fundamental in French society, where industry and positivism were developing rapidly. The debates on the place of science in education (recurrent throughout the century), and the low budgets devoted to higher education and research, echoed a declinist vision of French science, as opposed to a flourishing German science built on a model of higher education seen as extremely efficient. [40] This situation was exacerbated by France's defeat against Prussia in 1870. It was in this context that the *Association française pour l'avancement des sciences* and the *Société mathématique de France* were

39 « Mon excellent et bien regrettable ami M. Terquem affectionnait les mots *arithmologue, arithmologie* : c'est ce qui m'a décidé à donner à cet article et à d'autres qui le suivront, le titre qu'il porte et qui aurait été plus clair en ces termes : *Recherches élémentaires sur les nombres entiers.* »

40 This vision of a declining French science has been widely relayed by the actors and by historiography. Since the 1970s, several studies have put this situation into perspective: see [Weisz 1977] among others.

created in 1872; they quickly brought together in a differentiated and non-exclusive way the French and foreign mathematicians [Gispert 2015; 2002]. Each of these societies set up its own publication organ: the annual reports of the congresses for the *AFAS* and the *Bulletin* for the *SMF*.

While the curricula of *mathématiques élémentaires* and *spéciales* remained stable, the reforms of Minister Duruy and the Third Republic substantially transformed secondary education—with the development of special education and the establishment of secondary education for girls in particular—and higher education—with an increase in the number of teachers and students in the faculties, and the creation of *licence* and *agrégation* scholarships. Alongside Paris, higher education also developed in provincial faculties such as Bordeaux, Toulouse and Lille. This had consequences for the structure of the readership of the *NAM*. It was in 1888 that candidates for the *licence* and *agrégation* were explicitly targeted. Nevertheless, content intended for this type of readership had already been published for several years: the subjects of the *agrégation* since 1844 and articles related to the *licence* examinations since the 1860s [Ehrhardt 20??; Nabonnand & Rollet 2013].

The editorial landscape evolved accordingly, with the creation of new intermediate journals in France and abroad, such as in Germany or Belgium (see p. 64).[41] The multiplication of intermediate journals, and more generally the rise of the mathematical press, led to a fragmentation of the editorial landscape but also to the potential for new mathematical circulations, whether through articles, questions and answers or even bibliographic sections, as we will see through a few examples.

5.2. *Changes in Content and Form For* NAM *Headings*

Quantitatively, the number of entries and authors is significantly higher than in the first period, and the QA-section had acquired a dominant position. Two new author profiles were associated with the dynamism of this section: authors specialised in questions—such as Catalan and Lionnet[42]—or in answers—the high school teachers Moret-Blanc and Élie Fauquembergue, the artillery captain Charles Moreau or Auguste Morel, *répétiteur* in Sainte-Barbe. These four cases were quite different: Moret-Blanc answered many questions on various topics. Fauquembergue

41 As early as 1869–1870, high school journals were also created locally and had an ephemeral lifespan [Delcourt 2019].

42 As mentioned earlier, the case of Lionnet should nevertheless be taken with caution.

and Moreau were interested in questions of indeterminate analysis. The former participated in exchanges on cubic and biquadratic equations while the latter dealt with various questions with a good knowledge of Legendre's arithmetic statements for example. Morel, for his part, addressed questions typical of competitions, on divisibility and decimal fractions.

The proportion of students was also higher for this period, especially for published and especially unpublished answers (about thirty). Indeed, the answers of only seven students, five of whom were in higher education, were published. Responses by foreign students were also published: Hemming in 1865 in Zürich or Victor Schlegel, E. Kruschwitz and Eugen Otto Netto in Berlin between 1869 and 1871. This participation of Berlin students in the *NAM*, in the political context of the time, may come as a surprise; it is possible that Kummer or another professor from the University of Berlin encouraged them in this direction. Students could also publish articles, such as Fritz Hofmann in 1886, who was a student in Munich.

Compared to the first period, the number of reviews on arithmetic publications was much lower (6 over the period, including 3 on textbooks and 3 on Gauss' *Werke* and Lebesgue's treatises): the heading *Bibliography* was then mainly composed of lists of recent publications. These bibliographic lists informed readers of the publication of recent works but especially of the latest developments in the mathematical press. French and foreign research journals were predominant, while French intermediate journals such as *JME* or *JMS* were almost absent. The contents of the *NCM* were, however, announced, at least at the beginning. [43] A significant proportion of the authors of the referenced articles published in the *NAM*. [44] Unlike the previous period, no comments on the interest of number theory were developed.

Between 1863 and 1888, the examination reports published in the *NAM* showed that number theory was present in the *agrégation* program, with the same content as for *mathématiques élémentaires* and *spéciales*. The other arith-

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43 Note that this is consistent with the results obtained by Hélène Gispert in the case of the *Bulletin bibliographique* of the journal *L'Enseignement mathématique*: only one-third of the journals listed therein are related to teaching [Gispert 2018].

44 For example, for the year 1881, two articles were taken from AFAS reports (Desboves and Henry), one from the *NCM* (Realis) and two from the *Proceedings* of the Royal Society of London (Roberts), these four mathematicians also having published in the *NAM*. For the year 1886, of the twelve bibliographic entries, five were Cesàro's memoirs published in the *Annali di Matematica*, the *Rendiconti*, the *Annales de l'École normale*, the *Jornal de Sciencias maths...* and the *Giornale de Matematiche* and a note published in the *CRAS* by Jonquières. Several of Kronecker's memoirs published at the Berlin Academy were also listed.

metic exam subjects were from the Naval and Forestry School entry examinations, on divisibility questions and periodic fractions. [45] The subjects of the *Concours général* cited in the *NAM* included number theory questions at least eight times, mainly for the class of *Troisième* on divisibility. However, no arithmetic content was mentioned in the texts related to the certificates for the *licence.*

However, there were fewer articles on *mathématiques spéciales* and *élémentaires.* For example, only three articles were published on periodic fractions: two memoirs by Laisant and Étienne Beaujeux in 1868 and 1870 and one by Gustave de Coninck in 1874. Periodic fractions otherwise appeared in questions from Lionnet's treatise on elementary algebra and the associated answers. By way of comparison, the arithmetic content published in the *Journal de mathématiques élémentaires* (*JME*) and *Journal de mathématiques spéciales* (*JMS*) were closer to the *mathématiques élémentaires* and *spéciales* respectively, with articles and questions on divisibility, periodic and continued fractions. There were also more textbook reviews. This does not mean, however, that these journals were limited to content anchored in the curricula but the authors seemed to be more concerned about presenting mathematics adapted to the students. [46] In the *NAM,* the largely dominant mathematical theme of this period was indeterminate analysis, in a new way: the cubic and biquadratic indeterminate equations then had the main role, both in the articles and in the Q-A section (see part 5.3). A few other arithmetical themes were occasionally published. Some arithmetic articles were based on combinatorial theory. For example, Désiré André proposed an arithmetic formula in 1874 useful in number theory and combinatorics, particularly for students of *mathématiques spéciales* [André 1874, p. 188]. A second combinatorial example is the original demonstration of the quadratic reciprocity law proposed in 1872 by Egor Ivanovič Zolotarev, *Privatdocent* at the University of Saint-Petersburg [Boucard 2017, pp. 81–84]. This article was also one of

45 For example, in 1883, several elementary questions were asked in the Forestry School examination, such as: « Prove that the square of a prime number minus one unit is always divisible by 12 (except for the numbers 2 and 3) ».

46 Realis published, for example, a paper on the divisibility of arithmetic forms in 1885 in the *JMS* whose objective was the « popularisation of the principles of indeterminate analysis and number theory » to make them « within the reach of students ». These principles, although « excluded from the curriculum », belong to « the most elementary part of science » and can be useful « in different analytical questions » (« vulgarisation des principes de l'analyse indéterminée et de la théorie des nombres [...] à la portée des élèves [...] exclus des programmes d'enseignement [...] la partie la plus élémentaire de la science [...] en différentes questions d'analyse ») [Realis 1885].

the only ones to be directly related to statements by Gauss and Legendre (excluding indeterminate equations), which is a significant difference from the first period. The case of the *NCM* was different from this point of view: articles on the theorems of Fermat and Wilson for example were regularly published there, as well as papers on indeterminate equations by Cauchy and Libri from the 1830s and 1840s. Similarly, little was written about the use of complex numbers in number theory during this period. Finally, several entries dealt with mathematical questions circulating in several mathematical journals: the graphical approach to number theory (see part 5.4) as well as sums of similar powers and Bernoulli numbers. On this last point, Cesàro and Lucas in particular published several original memoirs in the *NAM* during the 1870s and 1880s. These works were part of a set of texts published in the *CRAS* and the *NCM* and assumed that the reader had knowledge of current research and methods such as symbolic calculation (see [Lucas 1877] for example).[47]

Compared to the first period, the role of the editors was much more discreet. Prouhet until 1867 and Gerono until 1887 were nevertheless more or less regular contributors to number theory texts in the *NAM*. The significant decrease in the number of arithmetic entries in the late 1880s can be explained by the death of several of the dominant authors (Lionnet in 1884, Realis and Moret-Blanc in 1886, Desboves in 1888, Genocchi in 1889, Gerono and Lucas in 1891) and by the fact that many turned away from the *NAM* to publish their works in other periodicals (Pépin in 1875, André in 1877 except for a question in the 1890s, de Jonquières in 1878, Lucas in 1881, Genocchi in 1885, Catalan and Desboves in 1886). One possible explanation is that the editors of the time, Brisse and Rouché, did not seem to have a particular affinity for number theory, unlike Terquem or Gerono for example.

5.3. *Research on Indeterminate Equations*

During this period, the indeterminate analysis of the first degree was only processed from questions on the number of solutions of given equations, as with Hermite in 1868 on the equation $x + y + z = N$. Catalan, Cesàro, then a pupil of the first in Liège, and Nicolas Goffart, a teacher in Paris, exchanged in the form of two questions and answers on the number of solutions of a set of indeterminate equations of the first degree between

[47] On arithmetic research around Bernoulli's numbers between 1850 and 1910, see [Destephen 2016]. Lucas' work on Bernoulli numbers and symbolic computation is also studied in [Décaillot 1999].

1882 and 1884 in the *NAM.* These two questions were in fact included in a broader set of texts on the same subject published between 1882 and 1890 in *Mathesis,* the *JME* and the *Mémoires* of the *Société des sciences de Liège* [Dickson 1919–1923, vol. II, pp. 66–68].

5.3.1. *The Indeterminate Equation* $x^3 + k = y^2$

A large part of the indeterminate analysis, however, concerned mainly cubic and biquadratic equations. [48] I do not propose here an exhaustive study on the corresponding works but prefer to focus on an example, that of the equations of the form $x^3 + k = y^2$. Seven articles related to this class of equations were published in the *NAM* between 1869 and 1885. This case is interesting to highlight some characteristics of the research of indeterminate analysis published in the *NAM.*

In 1869, Lebesgue published an article on the impossibility of three indeterminate equations. Lebesgue proposed a method based on appropriate transformations of the said equations. For example, he treated the equation $x^2 = y^3 + 7$ by dissociating two cases according to the parity of y: if y is even, the equation becomes $x^2 = 8v^3 + 7$ then, as x is odd, $8u + 1 = 8v^3 + 7$, which is impossible; if y is odd, the equation becomes $x^2 + 1 = (y + 2)[(y - 1)^3 + 3]$, where $(y - 1)^2 + 3$ is the form $4k + 3$ and therefore accepts a first divisor of the form $4b + 3$. Lebesgue deduced from this that the equation is impossible by implicitly using the properties of the divisors of the sums of two squares. [49] Lebesgue concluded his note with a question: « Can we demonstrate other special cases by the same method, without introducing imaginary numbers formed with the roots of the unit? » [Lebesgue 1869, p. 454] Then followed epistolary exchanges with Gerono who did not understand on which arithmetic property Lebesgue relied to conclude. [50] This exchange is interesting because it shows the complexity of the exchanges around the indeterminate equations. Lebesgue's demonstration was elementary, in the sense that it was based on results that a student could understand. However, it presupposed a certain arithmetic culture, particularly with regard to the properties of quadratic

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48 Alain Larroche (Institut de Mathématiques de Jussieu—Paris Rive Gauche, Université Pierre et Marie Curie) is currently working for his Phd thesis on the history of biquadratic Diophantine equations between 1875 and 1890.

49 Lebesgue used another method for his third example: the method of « non-congruence », already mentioned in 1847, which consisted in establishing the impossibility of an indeterminate equation from the impossibility of the corresponding congruence.

50 Archives de la Bibliothèque de l'Institut de France, MS 2031 (Bertrand), Correspondence from Gerono to Lebesgue.

divisors. The implicit use of these properties was not easy to grasp, even for Gerono who had been manipulating indeterminate analysis for several years and had been in regular epistolary contact with Lebesgue since at least 1864. Moreover, Lebesgue's final remark also informs us about the methods that seemed relevant to him for these indeterminate equations at the time: while he and other authors proposed reflections, albeit concise, on the use of complex numbers for number theory in the 1840s and 1850s, he was now looking for cases of indeterminate equations to solve without these same imaginary numbers.

On this link between complex numbers and indeterminate analysis, the case of Pépin is also interesting. In 1875 he published a paper in the *JMPA* in which he proposed a study of classes of complex integers of the form $a + b\sqrt{-c}$ (a, b and c integer) with an application to indeterminate analysis. He relied in particular on the theory of forms and Kummer's work on complex factors. He processed several sets of indeterminate equations including $x^2 + cn^{2\alpha} = z^3$ for particular values of c and thus obtained the solutions of $x^2 + a^2 = z^3$ for $a = 7, 9, 11$. Pépin published several times in the *NAM* at the same time but made no reference to his method or the equations $x^3 + k = y^2$.

Between 1876 and 1877, Brocard and Gerono published respectively in the *NCM* on the equation $x^3 + 17 = y^2$ (which admits the solution $x = 2$, $y = 5$) and in the *NAM* on the equation $x^3 = y^2 + 17$ already discussed by Lebesgue. In 1878, a memoir by de Jonquières also appeared in the *NAM*, where he chose a more general approach. De Jonquières noted that only special cases were in the case of the equation $x^3 \pm a = y^2$ and that they did not seem « to be connected to each other by any simple law » [Jonquières 1878, p. 374]. He therefore proposed a study of the same kind as Pépin's, namely a study on different values of a [Jonquières 1878, p. 376]. To do this, he developed a method similar to Lebesgue's, based on the shapes of the square sum divisors, without any comparison with Pépin's approach.

Realis, in 1883, published a paper on the equation $x^3 + k = y^2$ introduced by a bibliographical review of the articles published in the *NAM* on the subject. Realis noted that, with the exception of Pépin's memoir, these various works always dealt with cases of impossibility. On the contrary, his goal was to give examples of equations admitting solutions and to deduce additional arithmetic properties from them [Realis 1883, p. 289]. To do this, he relied on algebraic identity manipulations. For example, from the identity $(a^2 - 2b)^3 + b^2(8b - 3a^2) = (a^3 - 3ab)^2$, he obtained a solution for the case $k = b^2(\pm 8b - 3a^2)$. Thus, $2^3 - 4 = 2^2$ or $-3^3 + 52 = 5^2$. Moreover, with $b = 1$, we have $(a^2 \pm 2)^3 - (3a^2 \pm 8) = (a^3 \pm 3a)^2$ which allowed him to deduce that « any number of one of the forms $3a^2 \pm 8$ is the difference between a cube and a square » [Realis 1883, p. 290]. In the same volume, he asked a question on the same equation with other conditions on k, a question solved by algebraic manipulation by Fauquembergue in 1885, then by Brocard in 1891, again in the *NAM*. It seems to be the last time this class of equations was published in the *NAM* before 1916, when Hayashi proved the impossibility of $y^2 + 1 = z^3$ for y non-zero and $y^2 - 1 = z^3$ for $y^2 \neq 0, 1, 9$ in one of his memoirs on indeterminate equations.

However, this same class of equations was the subject of a series of articles in the 1890s, published in periodicals other than the *NAM*. Pépin proposed the resolution of a set of special cases of the equation in the *Memorie della Accademia Pontifica dei Nuovi Lincei* in 1892 and then in three notes published in the *CRAS* between 1894 and 1895. Fauquembergue gave incomplete evidence of the impossibility of $x^3 + 2 = y^2$ in *Mathesis* in 1896 and about ten articles concerned other particular cases in the *IM* between 1895 and 1904, including one by Jonquières in 1899 [Dickson 1919–1923, II, pp. 535–536]. Thus, if the *NAM* seemed to be the playground *par excellence* for this equation until the late 1880s, authors such as Pépin and Jonquières who published in the *NAM* turned to other periodicals.

5.3.2. *Some Characteristics of Work on Indeterminate Equations in the* NAM

First, the authors mentioned above dominated all publications on cubic and biquadratic equations in the *NAM*: Lebesgue, Jonquières, Realis and, to a lesser extent, Gerono and Pépin. Lucas and Desboves were also regular authors on this theme. Three approaches were distinguished: the treatment of a particular case of equation like $x^3 \pm 17 = y^2$, methods to show the impossibility of a set of equations [Lebesgue 1869] or on the contrary to solve a set of equations [Realis 1883].

The methods used were mainly elementary based on algebraic manipulations and well-known properties of quadratic divisors. For example, Pépin's thesis was known to several authors, but the tools used in it—theory of forms, integers and complex factors—had not been reinvested in the *NAM*. Globally, complex form and integer theory were absent from this work. However, several authors proposed different methods from those mentioned above: Lebesgue with his method of « non-congruence », Hermite with third order surface properties to solve the equation $x^3 + y^3 = z^3 + u^3$ [Hermite 1872] and Desboves with a Cauchy method using partial derivatives to treat homogeneous cubic equations with three unknowns of which at least one solution is known [Desboves

1886]. Forms of circulation appear via the questions or sets of texts, in the sense that the same equations were studied with explicit reference to previous publications. However, reappropriation of methods within the *NAM* seemed uncommon. For example, Lucas used Sylvester's residue method for the equation $x^3+y^3=Az^3$ but it was then not repeated by any author in the *NAM*. Sylvester, on the other hand, reproduced this memoir by Lucas in 1879 in the *American Journal of Mathematics*, created in 1878 and of which he was the editor, and then discussed it in an article inserted in the same volume [Sylvester 1879].

Compared to the first period, the links to research journals were of a different nature. The topicality of the indeterminate analysis of the third and fourth degree was published in the *NAM*—but also in the *NCM* or *Mathesis* for example—and in academic journals in Brussels, Liege, Paris or Roma. Some authors could react in the *NAM* on academic publications such as Prouhet in 1866 on a paper by Jordan published in the *CRAS* the same year to indicate that one of his results had already been obtained by Lebesgue in the 1830s. There were more authors who published both in the *NAM* and in research journals: in addition to Lebesgue and Catalan, Desboves, Pépin or Lucas for example.

More generally, a consultation of [Dickson 1919–1923, II] for cubic and biquadratic equations reveals that the journals most used by the authors in this context were *NAM* and *NCM*, then *Mathesis*, *ET*, then, at the end of the century, *IM*. Moreover, the disappearance of the *NAM* in the circulation space of the papers on the equation $x^3+k=y^2$ was more general: from 1888 onwards, there were only very occasional entries of indeterminate analysis (seven entries between 1893 and 1914). It should also be noted that there was almost no German journals, either intermediate or research journals, on this subject. Finally, these texts of the *NAM* remained references on the equation $x^3+k=z^2$. For example, Louis Joel Mordell in his 1914 memoir on the $x^3+k=y^2$ equation cited the work of Lebesgue, Gerono, Jonquières, Realis and Pépin [Mordell 1914, p. 60].[51]

5.4. *Mixture of Arithmetic and Geometry in the* NAM*: « Discrete Mathematics » that is rather Discreet*

Between 1860 and 1890, indeterminate analysis of the first degree was no longer the subject of articles in the *NAM*. However, some related papers were published in other media. For example, Camille de Polignac presen-

51 I would like to thank François Lê who has drawn my attention to Mordell's paper, see also Gauthier & Lê [2019].

ted to the *SMF* in 1877 a graphical method for solving and determining the number of integer and positive roots of the equation $ax + by = c$. This method was based on the consideration of an orthogonal lattice or « indefinite chessboard » [Polignac 1878, p. 158], and was reworked by Laquière the following year, again at SMF. These papers were following other papers on chessboards by Laisant and Lucas presented to the *SMF* in 1877 and 1878. Following Lucas, these different authors defined their approach as a « géométrie des quinconces » which constituted the « graphic painting of number theory » [Laquière 1879, p. 85].

Together with d'Ocagne and Catalan, these authors were part of a « discrete mathematics community » [Auvinet 2011, p. 299] identifiable from the 1870s onwards, and whose characteristic was to favour graphic visualisations to solve number theory problems in particular.[52] Lattices and chessboards were considered both as objects of study and tools to obtain and demonstrate arithmetic results. The *SMF* and then the *AFAS* were the privileged places for the diffusion of these original mathematics, and this, in a differentiated way. While the work on the « géométrie des quinconces » was presented in both societies, the *AFAS* congresses were an opportunity to present papers on mathematical recreation and, in Lucas' case, practical applications to textile issues with the « géométrie des tissus ». In his *Théorie des nombres*, Lucas also grouped these different researches under the name « géométrie de situation », a unifying domain for set of various problems linked to combinatorics, number theory, mathematical recreations and based on graphic methods [Lucas 1891].

In the *NAM*, on the other hand, the articles relating to graphic arithmetic were very few. In 1870 Lucas published a graphical method based on a chessboard to determine sums of similar powers from the first integers, that provoked an article on the same subject with the same method from high school teacher Édouard Amigues the following year. Laisant's only paper related to graphic arithmetic was in fact a reaction to a question asked by d'Ocagne in 1883 on the positions of a clock and which he had already asked in the *JME* [Auvinet 2011, p. 302]. However, at the 1887 AFAS congress he submitted a memoir entitled « Quelques applications arithmétiques de la géométrie des quinconces », in which he showed how to use this geometry to solve indeterminate equations of the first degree and to obtain properties on periodic fractions [Auvinet 2011, pp. 350–352]. The subject of the *Concours général* published in 1887 linked arithmetic and

52 On discrete mathematics, and especially Laisant, see the work of Jérôme Auvinet, including [Auvinet 2011; 2017]. Concerning Lucas, see Anne-Marie Décaillot's writings, including [Décaillot 1999; 2002].

geometry by studying the plane shared by an infinite number of squares numbered by integer coordinates. D'Ocagne gave a solution the following year but no article on graphic arithmetic followed. Only one very short note on the *géométrie des quinconces* was inserted by Lucas in the *NAM* in 1878 on the properties of the vertices of a chessboard, without any link with the curricula. Several authors of graphic arithmetic therefore published in the *NAM*, but mainly on other topics. However, this type of number theory was a priori adapted to the readers of the *NAM* by proposing an original approach with applications on subjects of the curricula.

Moreover, this kind of mathematics was not excluded from the intermediate mathematical press. In 1883 and 1888, Lucas asked questions about chessboards in the *JMS* and in 1883, the editor Gaston Albert Gohierre de Longchamps inserted there « Varieties » on Lucas' mathematical recreations. In 1882, the mathematics teacher Siegmund Günther published a paper entitled « Operative Arithmetik und Geometrie der Gittersysteme » in the *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, a journal to exchange ideas about the teaching of exact sciences in secondary schools. Günther had already published texts on mathematics and history of mathematics on themes linked to geometry of situation such as star polygons and polyhedra and magic squares. In his article, Günther summarised the work of French scholars in the field of « géométrie des quinconces », indicated their textile applications and compared them with the work of Gauss and Eisenstein on the use of geometric representations in number theory.

Mathematical recreations also had a limited place in the *NAM*, with only two articles on classic chess problems and a few announcements in the bibliographic section. The first article, by Bourget, contained a solution of the problem of the eight queens as an application of his earlier work on permutation theory [Bourget 1881, pp. 76–77]. The second article, by the previously mentioned student Hofmann, dealt with the knight's tour. He introduced complex integer coordinates to define the eight possible movements of the knight from a square and translated the condition of a closed tour into the nullity of the sum of directions of all movements [Hofmann 1886]. This graphic method developed by Hofmann resonated to some extent with his article on congruences published in 1882 in the *Mathematische Annalen* and based on graphic considerations relating to polygons with reference to Poinsot's work of 1845 mentioned above [Hofmann 1882].[53]

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53 The fact that Günther had begun teaching in Munich in the same year was perhaps not unrelated to this arithmetic publication in the *NAM*.

Authors such as Laisant or Lucas did not publish their writings on mathematical recreation in the *NAM*. Lucas' example is particularly significant. During this same period, Lucas presented recreational problems to SMF and AFAS. He published articles on this subject in the *NCM* and inserted a series of seven « Scientific recreations on arithmetic and geometry of situation » between 1879 and 1881 in the *Revue scientifique de la France et de l'étranger*. In the 1880s, Lucas also published several books and several dozen articles on mathematical recreation in a wide variety of journals: *La Nature*, the *Bulletin des sciences mathématiques et astronomiques*, the *Tablettes du chercheur* but also in the intermediate journals *Mathesis*, *JMS* and *JME*. Again, Lucas seemed to consider that mathematical recreation was an appropriate topic for intermediate journals, but none of his articles appeared in the *NAM*.

The previous analysis shows a number-theoretic content less adapted to the target readership than for the first period and a fragmentation of the elementary among different mathematical journals. Themes were present in all intermediate journals—periodic fractions, divisibility for example—but in different quantities and forms. On this point, the *NAM* contained very few articles and a few questions. With regard to graphic arithmetic, the actors seemed to favour publications associated with the *SMF* and then the *AFAS* even if some articles, dealing with indeterminate analysis or periodic fractions, responded a priori to the readership of the *NAM*. By contrast, many texts on quadratic, cubic and biquadratic equations appeared in the *NAM*. The methods presented were elementary, the examples treated allowed readers to practice particular forms of algebraic manipulation and reasoning on quadratic form divisors. However, they also assumed arithmetic knowledge that is not part of the mathematical background of the student reader of the *NAM*. To some extent, the arithmetic contents of the *NAM* seem less adapted to the objectives stated by the journal for this period: the number of textbook reviews was lower, the interest of number theory in mathematical training was no longer discussed and the majority of topics covered in the *NAM* were relatively far from the curricula. Between 1863 and 1888, the *NAM* was therefore a place of publication for original number theory research, whose themes differed in part from the number theory of the Academy of Sciences, the SMF or the AFAS [Décaillot 2007]: Bernoulli numbers were a common theme, but graphic arithmetic or work on the decomposition of numbers into prime factors (which was regularly presented to AFAS) was almost absent from the *NAM*.

6. 1895–1927: THE *NAM*, HIGHER EDUCATION AND NUMBER THEORY

The period[54] began with the reform of 1896, which changed the structure of higher education with the creation of provincial universities. These universities included the faculties as well as newly created technical institutes and laboratories. The undergraduate programmes were also transformed, and in particular adapted to students who had not necessarily passed through the classes of *mathématiques spéciales*, and new courses, sometimes more technical and practical, were introduced [Nabonnand 2006; Nabonnand & Rollet 2011]. At the same time, the editorial landscape was partially reconfigured: if the *JME* and *JMS* were then in their final days, new intermediate journals were created with the *Bulletin de mathématiques élémentaires* (*BME*), the *Bulletin de mathématiques spéciales* (*BMS*) and the *Revue de mathématiques spéciales* (*RMS*). With the internationalisation of mathematics at the end of the century [Parshall & Rice 2002], Laisant and Henri Fehr, of Geneva, created *L'Enseignement mathématique*, an international journal to promote collaboration between primary and secondary education stakeholders [Auvinet 2011; Gispert 2018]. In addition, new journals focused on question and answer exchanges also captured some of the potential readership of the *NAM*: *IM*, created by Lemoine and Laisant,[55] and the *Tablettes du chercheur, journal des jeux d'esprit et de combinaisons* (1890–1905), which contained a substantial amount of number theory.

For the *NAM*, this last period was characterised by a much smaller quantity and proportion of arithmetic articles. From 1896, the team of editors of the *NAM* was renewed and Laisant was part of it until his death in 1920 but, although he was at the head of several mathematical journals, he published only a few texts [Auvinet 2011]. Among the eight editors of the *NAM* during the period, only Raoul Bricard also published number theory texts. The two authors publishing more than ten entries, Bricard and Fontené, were not specialised in number theory. Nevertheless, other authors played a significant role in the circulation of number theory *via* the *NAM*, in different ways. For example, Cahen published several books on number theory, taught it at the *Faculté des sciences* in Paris for six years and was the author, with Karl Theodor Vahlen, of the chapter « The arithmetic theory of forms » in the *Encyclopédie des sciences mathématiques pures*

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54 Between 1889 and 1894, very few number theory entries were published in the *NAM*: 5 articles, 2 questions, 1 answer, as well as 3 elementary number theory topics for *agrégation* and *Concours général* and some announcements of recent publications.

55 Number theory was among the most represented fields in *IM* with geometry and the history of mathematics [Pineau 2006].

et appliquées, a major editorial project at the turn of the twentieth century [Gauthier 2007]. He regularly exchanged with Fontené—they both taught at *Collège Rollin* at the beginning of the century before Fontené became an academic inspector—and thus fostered arithmetical discussions in the *NAM*, especially on complex integers. André Gérardin, a bachelor of science and former student of the Faculty of Science of Nancy and the Sorbonne, published numerous articles on elementary number theory in intermediate journals at the beginning of the century, following in the footsteps of Lucas *et al.* He quickly took over the reins of the journal *Sphinx-Œdipe* (*SO*), created in 1906 and whose content became exclusively arithmetic from 1908. Gérardin used the *NAM* to feed his review and questions initially asked in the *NAM* and solved in *SO* were republished in the *NAM*.

6.1. *New Editorial Orientations for the* NAM *at the Turn of the Century*

For this period as for the previous one, few articles concerned elementary number theory adapted for high school students. [56] In addition, no author was identified as a secondary school student. Next to the *NAM*, the *BMS* and the *RMS* contained almost no arithmetic text. On the other hand, the *BME* regularly published arithmetical articles and questions related to its supposed readership, and therefore dealing with elementary properties of divisibility and prime numbers, periodic decimal fractions and some articles on themes outside curricula such as Wilson's theorem and Pell's equation. Laisant and Fontené at least published in both the *NAM* and the *BME*, obviously adapting the contents of their articles to the profile of both journals: Fontené did publish on divisors and periodic fractions in *BME* and algebraic integers in *NAM*. Similar themes could be treated in a different way in both journals: for example, Albert Lévy published an article on prime numbers in 1910 in the *BME*, indicating that he was dealing only with a special case of a result he had intended for the *NAM*.

This situation refers to a change in editorial policy intended by the new editors of the *NAM* in 1896 in connection with the above-mentioned university reforms: content for undergraduate and graduate students would be promoted, in order to « create a close link between Higher Education and the *Nouvelles Annales* » [Laisant & Antomari 1896, p. vi]

56 For example, around the turn of the century, they consisted essentially of an article by Lognon on Wilson's theorem, one by Bickmore on periodic decimal fractions in 1896, and a note by Bricard on a graphic demonstration of Fermat's theorem in 1903.

and more importance would be given to the previously neglected Q-A section in order to best serve the readers. Several arithmetic themes treated during this period were in this direction: arithmetic of specific complex number classes ([Autonne 1907; Pasquier 1918] for example), geometric approaches to number theory (p. 46) indeterminate equations of degree greater than 2 (p. 48). In addition, the analysis of the corpus highlights the multiplicity of relationships between the arithmetic content of *NAM* and higher education.

6.2. *New References on Number Theory*

First, new literature in number theory was published at the turn of the century, sometimes in connection with courses taught in higher education institutions in Paris: Tannery at the *École normale supérieure* in 1891–1892, Cahen at the *Faculté des sciences* between 1909 and 1914 thanks to an anonymous grant, or Châtelet at the *Collège de France* in 1911 at least [Goldstein 2009]. In the reviews, comments were regularly made about the importance of number theory and its applications in other mathematical fields and on France's backwardness in this field. As in the 1850s and 1860s, the absence of recent treatises on number theory in French was highlighted: for example, Cahen's *Éléments de théorie des nombres* (1900) was welcomed with relief because it filled « in the French mathematical literature of our time, a regrettable gap from two points of view » since the other French treatise devoted to number theory—the *Théorie des nombres* by Legendre—« has grown old in many ways and no longer meets current requirements » [Bricard 1900, p. 476]. For Bricard, Cahen's book was useful for two kinds of readers:

> The book of Mr. Cahen perfectly fulfills this double program: on one hand, to expose parts of arithmetic that are essential for the full understanding of other theories; on the other, to be accessible to the most unprepared readers, not using any notion previously acquired. [Bricard 1900, pp. 477–478] [57]

Algebraic number theory was regularly cited as an important part of number theory and almost absent from French publications as suggested by Bricard's commentary on Cahen's *Théorie des nombres*: « We look forward to the part devoted to algebraic numbers and ideals with particular anticipation. » [Bricard 1913a, pp. 470–471] In addition to general comments,

57 « Le livre de M. Cahen remplit parfaitement ce double programme : d'une part, exposer les parties de l'Arithmétique qui sont essentielles pour l'intelligence complète d'autres théories ; de l'autre, être accessible aux lecteurs les plus dénués de préparation, en ne faisant appel à aucune notion antérieurement acquise. »

the reviewers briefly described the content of the books and indicated original methods and results that might be of interest to the readers of the *NAM*. Bricard indicated, for example, the interest of the geometric demonstrations given by Cahen in his *Théorie des nombres* [Bricard 1913a].

The number theoretic books identified constituted new references for the authors and readers of the *NAM* and were at the origin of arithmetic exchanges. Bricard referred, for example, to Tannery's lectures in his article on the quadratic nature of 3 [Bricard 1897]. Cahen's *Éléments* were also quoted several times, by Fontené and Cahen himself at least, during their exchanges on certain classes of complex integers and the associated arithmetic [Cahen 1903; Fontené 1903].

6.3. *The* NAM *and Higher Education*

From 1896 onwards, higher education appeared in different forms in the *NAM*. In addition to the above-mentioned reviews, collaborations between editors and university professors allowed the publication of lists of *licence* certificates or courses given: in 1898, the University of Montpellier offered a certificate in higher algebra, with a course on field and Galois theory; in 1902, titles of American university courses were published, such as a course by Fite on group and number theory at Cornell University. The academic orientation of the *NAM* was reaffirmed at the end of the war. The examination for the *agrégation* and suggested answers were included in the last volumes of the *NAM*. Diophantine analysis seemed to have a less elementary place: a geometry problem concluded by an introduction to Diophantine equations in 1923 (V, 2, 107–120) and the properties of order cycles n in 1925 (VI, 1, 366).[58] Élie Cartan, then a professor at the Faculty of Sciences in Paris, proposed a general solution to the problem in 1927, for a general numbering system [Cartan 1927].

Then, of the twelve students publishing in the *NAM* over this period, none were identified as high school or college students: they studied at the *ENS*, *Polytechnique*, *Centrale* or Paris' science faculty. In addition to answers, they also published articles on number theory topics that were more advanced than previously. For example, Paul was still at the *ENS* when he published his article on imaginary integers and their geometric representation in 1906. Foreign students were also well represented, reflecting a more general situation: Michel Petrovitch, Trajan Lalesco and

[58] A cycle of order n of a number N is the set of n-digit numbers obtained by rotating circularly, in all possible ways, the digits of N, even if it means adding zeros if $n > k$. For example: the cycle of order 3 of 58 is the set: 058, 580, 805.

Georges Tzitzeica completed their theses in Paris before obtaining an academic position in their home country (in Belgrade for Petrovitch and Bucharest for Lalesco and Tzitzeica) [Gispert 2015, pp. 139–143]. They could publish in the *NAM* during their Parisian studies—Lalesco on quadratic forms [Lalesco 1907] before his thesis in 1908—or after—Petrovitch, doctor since 1894, published in the *NAM* from 1896, including an article on the geometric distribution of prime numbers in 1913 [Petrovitch 1913]. Students also published from abroad: Michael Bauer of Budapest on finite group theory in algebra and congruences in number theory between 1900 and 1902, René Goormaghtigh of Belgium, on indeterminate equations, between 1913 and 1918 [Bauer 1900; 1902; Goormaghtigh 1916].

Finally, among the authors of the 14 theses defended in Paris between 1900 and 1914 [Leloup 2009, p. 54], some published in the *NAM*, such as Gaston Cotty or Léon Pomey. Cotty, with a 1912 doctorate, published a paper on arithmetic geometry in the *NAM* which was close to his thesis, and related to properties of abelian functions and quadratic forms. Cotty recalled on this occasion the interest of geometric approaches in various fields of mathematics:

> The correspondences between algebraic and geometric beings abound in all branches of mathematics; even if they do not constitute a means of discovery, they are nevertheless very useful in making a whole set of results more often than not intuitive, quite difficult to establish analytically and in allowing certain proposals to be linked to each other, whose relationships did not appear otherwise. [Cotty 1913, p. 206] [59]

6.4. *A Variety of Geometric Approaches to Number Theory*

This quote from Cotty is interesting because it characterises several papers published in the *NAM* based on a geometric approach to number theory. As we have seen previously, Laisant was precisely one of the promoters of graphic arithmetic and repeatedly stressed the importance of visualisation processes in mathematics education. For example, he included a chapter on *géométrie de situation* in his collection of problems for *mathématiques spéciales*, including questions on chessboards and magic

59 « Les correspondances entre êtres algébriques et êtres géométriques abondent dans toutes les branches des mathématiques ; lors même qu'elles ne constituent pas un moyen de découverte, elles sont cependant fort utiles en rendant le plus souvent intuitifs tout un ensemble de résultats assez difficiles à établir analytiquement et en permettant de rattacher les unes aux autres certaines propositions dont les rapports n'apparaissaient pas autrement. »

squares [Laisant 1893]. At the beginning of the century, he also participated in many debates on the place of science in secondary education. In this context, the importance of visualisation and intuition in mathematics was emphasised by Laisant and Poincaré, in particular [Auvinet 2011; Gispert 2007]. As before, however, the editorial niche for chessboards and mathematical recreation was elsewhere: against eight entries in the *NAM* over the entire period, magic squares, graphic arithmetic and elementary number theory were at least the subject of about forty articles and reviews in the *EM*, also edited by Laisant, between 1899 and 1911. In the *NAM* were mainly published four articles by Gaston Tarry [60] and two by Bricard. The latter used, for example, a chessboard to study the properties of the residues of multiples of an integer, and introduced the notion of a quadratic chessboard, which allows visualisation of quadratic residues for a given modulus.

Another geometric treatment of number theory was introduced at the end of the century *via* several translations by Léonce Laugel. Former embassy representative, Laugel translated many mathematical texts from German at the turn of the century. [61] Klein's and Minkowski's translated papers dealt respectively with a geometric approach to continued fractions and the geometry of numbers. Minkowski's text [Minkowski 1896] was the result of a lecture presented at the Chicago Congress in 1893 and was therefore intended for an audience of mathematicians but not necessarily specialists in number theory: it was therefore an introduction to Minkowski's work in which geometric representations were central to the contexts of mathematical discovery and communication [Gauthier 2007]. These translations provoked reactions in the *NAM*: the engineer Georges Husquin of Rhéville published on continued fractions in 1897 following Klein [Husquin de Rhéville 1898]; and Bricard and Cahen returned to Minkowski's geometry of numbers for a demonstration of the theorem of two squares for example [Bricard 1913b]. As in the work on graphical arithmetic, visualisation in number theory was considered fundamental here, but at a higher mathematical level.

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60 Tarry's research is more generally in line with the themes addressed by Laisant or Lucas for example: see [Barbin 2017].

61 The database of *Jahrbuch* lists about twenty translations by Laugel, mainly published in the *NAM*. He was also the author of translations for *JMPA* and of Riemann's *Werke* [Chatzis et al. 2017, p. 30]. An article by Sébastien Gauthier and Catherine Goldstein on Laugel is in preparation.

6.5. *Indeterminate Equations Between 1915 and 1918*

The texts on indeterminate equations published before 1915 were for the most part continuous with work done in the previous period: Gérardin and Pépin for example relied on elementary methods by referring to previous publications. One exception was Dmitry Mirimanoff, who published two articles in 1903: in the first, he quoted Hermite's geometric method [Hermite 1872] applied to the equation $x^3 + y^3 + y^3 + z^3 = t^3$ to give a more direct one [Mirimanoff 1903a]; in the second, he used the theory of elliptic functions for the equation $(x + 1)^l - x^l - 1 = 0$.[62] To some extent, this inaugurated the use of non-elementary methods for indeterminate analysis in the *NAM*, echoing its more academic orientation.

Over the period 1915–1918 in France, publications on indeterminate equations were mainly concentrated in the *NAM*, *IM* and *SO*. Gérardin, Raymond Alezais, Laisant and Joseph Joffroy published articles in these three journals on indeterminate equations, based on elementary methods. In 1918, Émile Turrière noted that a geometry problem treated by Gérardin in terms of indeterminate equations could be solved through geometric applications of elliptic functions. Two other authors published papers on indeterminate equations over this period, using non-elementary geometric methods. In his three memoirs published between 1910 and 1918 [Hayashi 1910; 1916; 1918], Hayashi took up old problems treated in the *NAM* in the second period such as the properties of consecutive integers or the equation $x^3 - 1 = y^2$. Quoting the work of Gerono, Lebesgue and others from the 1860s to 1880s, he nevertheless adopted an approach in terms of integer and rational coordinates of cubic curves. The second author was Edmond Maillet, professor of analysis at the *École des Ponts-et-Chaussées*, and a specialist in number theory. Since the late nineteenth century, his work had included indeterminate equations, among them Fermat's last theorem, and was based on several methods, including recurrent series and ideals [Dickson 1919–1923; Goldstein 1999]. His two memoirs published in the *NAM* in 1916 and 1918 were based on the work of the Norwegian Axel Thue on approximations of algebraic numbers and its applications to Diophantine analysis published in the *JRAM* in 1909 [Goldstein 2015]. Here Maillet studied certain classes of indeterminate equations admitting a finite number of solutions geomet-

62 Born in Russia, Mirimanoff stayed in Paris at the end of the 19th century where he attended Hermite's and Poincaré's classes, among others before obtaining the rank of doctor in 1900 and becoming *Privatdozent* in Geneva in 1901 [de Rham 1980]. From 1907, next to research journals such as the *CRAS* or the *JRAM*, he used the *EM* to disseminate his research on indeterminate analysis.

rically, and proposed a « really practical way of obtaining an upper bound for the modulus of these solutions » [Maillet 1918, p. 281] [63] to obtain a complete solution of these equations by a finite number of tests. These two examples illustrate the contrast with the previous period, in solving similar problems.

Between 1919 and 1927, only twelve arithmetic entries were published in the *NAM*. In general, the journal was experiencing editorial difficulties and was struggling to find a formula adapted to its readership. The war was particularly deadly for scientific elites. Between 1914 and 1925, the number of mathematical publications listed in the *Jahrbuch* fell significantly, suggesting a general decrease in mathematical publications [Aubin & Goldstein 2014]. Number theory, with algebra, was probably neglected all the more in the 1920s, particularly because young and promising number theorists had died—such as Cotty and Lambert for example—and because others directed their work towards new, more applied research themes. One can also observe a virtual absence of number theory in the *CRAS* and the *JMPA* [Goldstein 2009].

7. CONCLUSION

This analysis shows that number theory appears as both multiple and variable within the *NAM*. The three periods studied each have their own characteristics. Until 1862, the *NAM*, under the ægis of Terquem and Lebesgue, appeared as a place to initiate and promote number theory. This field was presented as ideal for training minds and selecting the best students. In practice, arithmetic concepts were introduced to simplify and give more general, direct methods on teaching subjects. Objects and arithmetic statements were studied for their own interest in order to initiate readers, while no French and current treatises on number theory were available. Between 1863 and 1888, many publications dealt with current and original research on cubic and biquadratic indeterminate equations based on elementary methods. These exchanges were led by regular authors of the *NAM* who also invested in other journals, intermediate or not. This was the case, for example, for Lucas, Realis or Catalan. The mentions of teaching were in contrast rare. Finally, after 1895, the general academic orientation decided on by the editors of the *NAM* was very strongly felt in the case of number theory, both in terms of content and authorship.

63 « moyen réellement pratique d'avoir une limite supérieure du module de ces solutions ».

Over the entire period of existence of the *NAM*, indeterminate analysis remained the main theme. It was regularly investigated « in the students' colour », either in the form of indeterminate equations of the first degree, or because the equation to be solved or the decomposition to be obtained was based on simple algebraic manipulations. In this sense, it responded to the target audience of the *NAM*. In addition, in most cases, the methods used were in the lineage of Euler, Lagrange and Legendre. Thus, if decompositions into sums of squares or cubes were studied, Gauss' theory of quadratic forms remained completely absent from the *NAM*. This echoed to some extent some of the French arithmetical work of the first part of the nineteenth century, valuing an approach to number theory through equations. It was only during the third period that different types of methods—algebraic, geometric, analytical to a lesser extent—coexisted, thus inducing a separation between elementary and non-elementary content.

Through the *NAM*, some authors insisted on the existence of links between different domains. On the one hand, mainly during the first period, these links existed between algebra and number theory with circulation of methods between algebraic theory of equations and indeterminate equations or congruences. Authors such as Terquem and Lebesgue regularly stressed the interest of number theory in algebraic questions. They also discussed the use and properties of complex number classes, in analogy with the complex roots of algebraic equations, or in the context of number theory theorems, such as the two square theorem or Fermat's last theorem. These objects were again at the heart of several articles in the third period, when algebraic number theory was in full expansion: the questions then consisted in determining the arithmetic properties of these number classes. Another original disciplinary configuration mixed number theory, with geometry and eventually algebra. Terquem introduced it via Poinsot's work and then some graphic arithmetic work was inserted into the review in the second period. It was especially from the 20th century onwards that multiple geometric approaches were presented in the *NAM*, certainly in relation to the educational and epistemological context of the time. On the other hand, analysis remained practically absent throughout the period in arithmetic publications, with the exception of a few references to the theory of elliptic functions.

With number theory, we find aspects already known about *NAM*, such as the specific role of some publishers as Terquem, and the fact that published content often went beyond curricula or the optimal functioning of the Q-A section during the second period. However, there are a few characteristics that should be highlighted. First of all, for number theory, the proportion of teachers and of their publications is much higher than for the contents of *NAM* as a whole. This field therefore seems to draw a specific structure for the authorship. Further, the second period appears to have been unique in relation to the other two. Some themes such as the use of complex numbers in number theory or geometric approaches were absent or very much in the minority compared to other journals during the second period. Finally, the articles on cubic and biquadratic indeterminate equations published in *NAM* seem to partially contradict the idea that the second period would have been the one most consistent with the goals of the journal. Indeed, the students participated very little in these exchanges which, on the one hand, were foreign to the classes of *mathématiques spéciales* and on the other hand, often required elementary but uncommon arithmetic background. A precise study of the exchanges around Bernoulli numbers, involving some of the same authors, would probably lead to the same conclusion.

Based on *NAM* and number theory, some (non) circulations in the editorial space have also been highlighted. During the first period, we found well-known circulations between French and German journals: the editors of the *NAM* (and *JMPA*) include in their respective journals translations of memoirs from *JRAM* or Berlin academic memoirs. However, I didn't find any trace of the *Archiv* in the *NAM*. On the other hand, articles published in the *JMPA* or the *NAM* were regularly summarised by Grünert, the editor of the *Archiv* and some authors of this journal reacted quickly to the content of the *NAM*. During the second period, German journals were practically absent in the exchanges on indeterminate analysis, except for the bibliographic sections. Arithmetic research circulated between French and Belgian journals and, to a lesser extent, Italian and British ones, reflecting the international structure of the mathematical community of the *AFAS* [Gispert 2002]. During this same period, there was a fragmentation of the so-called elementary number theory contents within the editorial landscape: graphic arithmetic or number factorisation had a very limited place in the *NAM* whereas these themes were well represented in the *AFAS* in particular. While *NAM* was one of the main journals in which to publish on indeterminate analysis until the 1880s, the authors stopped using this journal to publish on this topic from the 1890s onwards, in favour of Belgian journals and the *IM* in particular. Finally, the content directly related to curricula decreased from the second period onwards, probably in favour of *JME* and *JMS* at first, then *IM* and *EM* afterwards. These few elements encourage us to clarify these comparisons, which are certainly instructive.

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8. APPENDICES

8.1. *List of Mathematical Journals Used in this Paper*

8.1.1. *Journal for Teachers and Students*

- **Belgium** [64]
  - *Nouvelle correspondance mathématique* (1875–1880): *NCM*
  - *Mathesis* (1881–1965)
- **France**
  - *Journal de mathématiques élémentaires* (1877–1879 then 1882–1901): *JME*
  - *Journal de mathématiques élémentaires et spéciales* (1880–1881): *JME*
  - *Journal de mathématiques spéciales* (1882–1897): *JMS*
  - *Bulletin de mathématiques élémentaires* (1895–1910): *BME*
  - *Bulletin de mathématiques spéciales* (1894–1900): *BMS*
  - *Revue de mathématiques spéciales* (1890–1998): *RMS*
  - *L'Enseignement mathématique* (1899-...): *EM*
- **Germany**
  - *Archiv der Mathematik und Physik* (1841–1920): *Archiv*
  - *Zeitschrift für Mathematik und Physik* (1856–1917)
  - *Zeitschrift für Mathematischen und Naturwissenschaftlichen Unterricht aller Schulgattungen* (1870-...)

8.1.2. *Other Mathematical Journals*

- *Bulletin de la Société mathématique de France* (1872-...): *BSMF*
- *Compte-rendus de l'Académie des sciences de Paris* (1835-...): *CRAS*
- *Compte-rendus de l'Association française pour l'avancement des sciences* (1873–1914) : *AFAS*
- *Intermédiaire des mathématiciens* (1894–1926): *IM*
- *Journal de mathématiques pures et appliquées* (1836–1934): *JMPA*
- *Journal für die reine und angewandte Mathematik* (1826-...): *JRAM*
- *Sphinx-Œdipe* (1906–192?): *SO*

64 Pauline Romera-Lebret is currently conducting research on Belgian mathematical journals. Some of her results were presented in 2015 [Romera-Lebret 2015].

8.2. *Professional Categories of Authors*

The authors of unpublished responses are included. For the counts, when an author occupied several professions during the period under consideration, the one that appeared most often was chosen. For the other items, the number of entries is indicated.

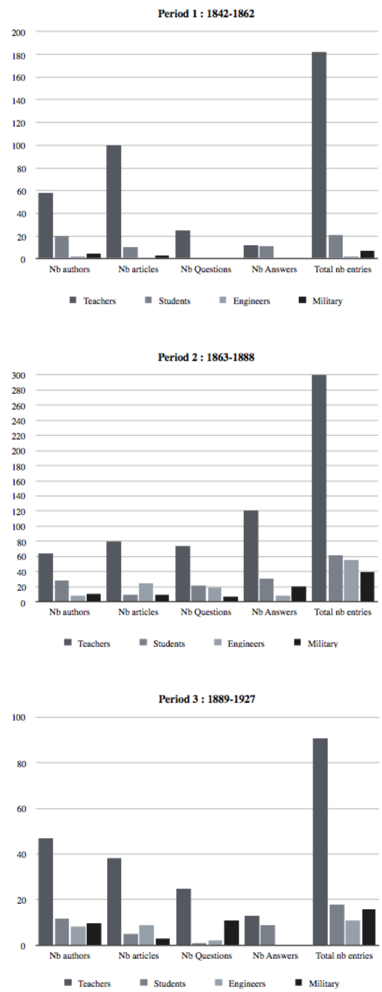

FIGURE 1. Professional categories of authors.

Table 4. Number of entries from authors who have published more than ten times or more than four articles related to number theory in the NAM in the form of e (a, q, r) where e represents the total number of entries, a the number of articles, q the number of questions and r the number of answers. The occupations indicated correspond to those held during the publication period(s).

Name	Profession	1842–1862	1863–1888	1889–1927	**Total**	% NT
Terquem, O.	Teacher & Librarian at the central repository of artillery	48 (13/0/0)			48	13,3%
Realis, S.	Student (Paris, 1843) then engineer (Torino)	1 (1/0/0)	42 (18/16/5)		44	50%
Lionnet, E.	Highschool teacher	5(4/1/0)	37 (5/28/3)		42	61,8%
Lebesgue, V.-A.	University teacher (Bordeaux)	24 (21/2/1)	12 (2/6/3)		36	46,8 %
Catalan, E.	Repeater at the *École polytechnique* then university teacher (Liège)	11 (7/3/0)	24 (2/12/3)		35	31,5%
Lucas, É.	Military (1870) then highschool teacher		32 (23/8/1)		32	57,1%
Moret-Blanc, C.	Highschool teacher (Le Havre)		29 (5/1/20)	1 (0/0/1)	30	19%
Cesàro, E.	Student then university teacher (Liège)		28 (7/19/2)		28	28%
Gerono, C.	Highschool teacher	7 (4/0/1)	10 (7/1/1)		17	11,6%
Laisant, C.-A.	Military (Brest), politician then Repeater at the *École polytechnique*		10 (4/4/1)	8 (1/2/3)	17	19,4%
Prouhet, E.	Repeater at the Polytechnic School	8 (5/1/2)	6 (0/0/0)		14	6,2%
Bricard, R.	Teacher at *École centrale*			13 (4/3/1)	13	7,7%
Fontené, G.	Highschool teacher then academic inspector			12 (6/0/5)	12	6,6%
André, D.	Highschool teacher		11 (7/1/3)		11	44%
Brocard, H.	Military		6 (1/3/2)	5 (0/0/5)	11	8,5%
Desboves, H.	Highschool teacher	1 (1/0/0)	8 (4/2/0)		9	33,3%
Genocchi, A.	Attorney then university teacher	6 (4/0/2)	3 (1/0/0)		9	20,5%
Weill, I.-M.	Highschool teacher		3 (3/0/0)	4 (3/1/0)	7	16,3%
de Jonquières, E.	Military		6 (4/0/0)		7	11,9%
Guibert, M.-P.-A.	Examiner at the Naval School	4 (4/0/0)			4	80%
Henry, C.	Librarian		4 (4/0/0)		4	100%
Tarry, G.	Contribution controller (Alger)			4 (4/0/0)	4	36,4%

8.3. *List of Reprints and Translations of Number Theory Published in* NAM

List of Translations and Reprints in the NAM (1842-1927)								
Original Text					Version in *Nouvelles annales de mathématiques*			
Year	Author	Original Title	Journal	Language	Year	Translator	Title	Type
1801	Gauss, Carl Friedrich	Disquisitiones arithmeticae	X	Latin / French	1843	Terquem, Olry	Théorème de Wilson d'après M. Gauss	Reformula-tion
1828	Dirichlet, Johan Peter Gustav Lejeune	Démonstrations nouvelles de quelques théorèmes relatifs aux nombres	Journal de Crelle	French	1845	Terquem, Olry	Généralisation de la théorie des nombres associés et théorèmes y relatifs. D'après M. Lejeune-Dirichlet	Reformula-tion
1828	Clausen, Thomas	Die Function $1/a\ldots$ durch die Anzahl der a ausgedrückt	Journal de Crelle	German	1846	Terquem, Olry	Détermination de la fraction continue périodique à un terme, en fonction du nombre des fractions ; d'après M. Clausen (Th) d'Altona	Translation & reformuation
1833	Scherk, Heinrich Ferdinand	Bemerkungen über die Bildung der Primzahlen aus einander	Journal de Crelle	German	1847	Terquem, Olry	Formation des nombres premiers les uns par les autres, d'après le professeur Scherk	Summary
1835	Jacobi, Carl Gustav Jacob	Über den Steinerschen Satz von den Primzahlen im 4. Hefte des 13. Bandes dieses Journals	Journal de Crelle	German	1848	Terquem, Olry	Théorème arithmologique de M. Steiner, démontré par M. Jacobi	Translation & reformulation
1810	Poinsot, Louis	Mémoire sur les polygones et les polyèdres	Journal de l'École polytechnique	French	1849	Terquem, Olry	Sur les polygones et les polyèdres étoilés, polygones funiculaires ; d'après M. Poinsot	Summary

Year	Author	Original Title	Journal	Language	Year	Translator	Title	Type
1844	Eisenstein, Gotthold	Transformations remarquables de quelques séries	Journal de Crelle	French	1849	Terquem, Olry	Sur les fractions continues, par M. G. Eisenstein	Selection
1835	Stern, Moritz Abraham	Beweis dreier Lehrsätze, mitgetheilt von Steiner, Bd. 13. S. 361 u. 362, nebst zwei anderen Aufgaben	Journal de Crelle	German	1849	Terquem, Olry	Sur trois théorèmes arithmologiques de M. Steiner, démontrés par M. Stern	Translation & modif.
1859	Lebesgue, Victor-Amédée	Exercices d'analyse numérique	X	French	1849	Terquem, Olry	Extrait des exercices d'analyse numérique ; par M. Lebesgue	Selection & reformulation
1849	Bertrand, Joseph	Traité d'arithmétique	X	French	1849	Terquem, Olry	Questions sur la numération et sur le plus grand commun diviseur	Selected questions
1844	Eisenstein, Gotthold	Allgemeine Auflösung der Gleichungen von den ersten vier Graden	Journal de Crelle	German	1849	Lebesgue, Victor-Amédée	Résolution générale des équations des quatre premiers degrés ; par M. P.-G. Eisenstein	Translation
1845	Kronecker, Leopold	Beweis daß für jede Primzahl p die Gleichung $1+x+x^2+\cdots+x^{p-1}=0$ irreductibel ist	Journal de Crelle	German	1849	Terquem, Olry	Nouvelle démonstration de l'irréductibilité de l'équation $1+x+x^2+\cdots+x^{p-1}=0$; p étant un nombre premier ; D'après M. L. Kronecker	Translation & reformulation
1835	Zornow, A.-R.	De compositione numerorum e cubis integris positivis	Journal de Crelle	Latin	1850	Terquem, Olry	De la composition des nombres en cubes entiers et positifs ; d'après M. Zornow	Partial translation & summary

Year	Author	Original Title	Journal	Language	Year	Translator	Title	Type
1850	Heine, Eduard	Über die in der Gaußischen « Summatio quarumdam serierum singularium » vorkommenden Reihen	Journal de Crelle	German	1850	Terquem, Olry	Sur la formation de deux séries qu'on rencontre dans la dissertation suivante de M. Gauss : *Summatio quarumdam serierum singularium* ; d'après M. le professeur E. Heine, de Bonn	Translation & reformulation
1840	Jacobi, Carl Gustav Jacob	Elementarer Beweis einer merkwürdigen analytischen Formel, nebst einigen aus ihr folgenden Zahlensätzen	Journal de Crelle	German	1850	Terquem, Olry	Théorème de Jacobi, énoncé dans le *Journal de Crelle*, t. 13, 1840	Selected translation
1850	Thacker, A.	Ein Beitrag zur Zahlentheorie	Journal de Crelle	German	1851	Terquem, Olry	Solution d'un problème sur la sommation d'une somme de puissances	Translation
1741	Euler, Leonhard	Solutio problematis ad geometriam situs pertinentis	Commentarii academiae scientiarum Petropolitanae	Latin	1851	Coupy, Émile	Solution d'un problème appartenant à la géométrie de situation, par Euler	Translation
1835	Göpel, Adolphe	De æquationibus secundi gradus indeterminatis	Thèse	Latin	1853	Terquem, Olry	Sur l'extension d'un théorème de Legendre et d'un théorème de Fermat	Translation & Summary
1843	Euler, Leonhard	Lettre XXXIX, Euler à Goldbach ; 6 mars 1742	Correspondance mathématique et physique de quelques célèbres géomètres du XVI[e] siècle	German	1853	Terquem, Olry	Théorème sur la somme de deux carrés d'après Euler	Selected translation

Year	Author	Original Title	Journal	Language	Year	Translator	Title	Type
1848	Hermite, Charles	Note de M. Hermite	Journal de mathématiques pures et appliquées	French	1853	Terquem, Olry	Un théorème de Fermat	Reformulation
1854	Borchardt, Carl Wilhelm	Application des transcendantes abéliennes à la théorie des fractions continues	Journal de Crelle	French	1854	Terquem, Olry	Théorie des fractions continues algébriques	Summary
1854	Dirichlet, Johan Peter Gustav Lejeune-	Über ein die Division betreffendes Problem	Journal de Crelle	French	1854	Terquem, Olry	Sur les résidus dans la division arithmétique	Summary
1846	Jacobi, Carl Gustav Jacob	Über die Kreistheilung und ihre Anwendung auf die Zahlentheorie	Journal de Crelle	German	1856	Laguerre-Werly, Edmond Nicolas	Sur la division du cercle et son application à la théorie des nombres	Translation & comments
1816	Gauss, Carl Friedrich	Methodus nova integralium valores per approximationem inveniendi	Commentationes Societatis Regiae Scientiarum Gottingensis recentiores	Latin	1856	Terquem, Olry	Sur les fractions continues algébriques ; d'après Gauss	Selected translation
		Initial text not found			1859		Deux théorèmes de maximums arithmologiques. D'après M. Oettinger, à Friburg	
1859	Bellavitis, Giusto	Sulla partizione dei numeri e sul numero degli invarianti	Annali di matematica pura ed applicata	Italian	1859		Bibliographie de la partition des nombres	Selection

Year	Author	Original Title	Journal	Language	Year	Translator	Title	Type
1859	Dirichlet, Johan Peter Gustav Lejeune. Hoüel, Jules, traducteur	Sur le caractère biquadratique du nombre 2. Extrait d'une lettre de M. Dirichlet à M. Stern	Journal de mathématiques pures et appliquées	Allemand / French	1862	Hoüel, Jules	Arithmologie. Sur le caractère biquadratique du nombre 2	Extract
1868	Catalan, Eugène	LXVI Sur la partition des nombres (octobre 1867)	Mélanges mathématiques	French	1869	Catalan, Eugène	Note sur la partition des nombres	Reprint
1870	Bills, S.	Question 3159 (proposed by A. Martin)	Mathematical Questions with their Solutions from the «Educational Times»	English	1871	–	Problème d'algèbre	Translation
1867	Catalan, Eugène	Rectification et addition à la «Note sur un problème d'analyse indéterminée»	Atti dell'Academia pontificia de' nuovi lincei	French	1867	Catalan, Eugène	Rectification et addition à la «Note sur un problème d'analyse indéterminée»	Reprint
1867	Catalan, Eugène	Note sur un problème d'analyse indéterminée	Atti dell'Academia pontificia de' nuovi lincei	French	1867	Catalan, Eugène	Note sur un problème d'analyse indéterminée	Reprint
1873	Catalan, Eugène	Recherches sur quelques produits indéfinis	Mémoires de l'Académie royale des sciences, des lettres et des beaux-arts de Belgique	French	1874	Catalan, Eugène	Propositions relatives à la théorie des nombres	Extract

Year	Author	Original Title	Journal	Language	Year	Translator	Title	Type
1880	Govi, Gilbert	Nota del Socio Ordinario Gilberto Govi (Adunanza del 5 Giugni 1880)	Rendiconto dell'Accademia delle scienze fisiche e matematiche	Italian	1880	Marre, Aristide	Sur quelques lettres inédites de Lagrange publiées par M. Balthasar Boncompagni	Translation
1895	Klein, Felix	Ueber eine geometrische Auffassung der gewöhnlichen Ketten-bruchentwickelung	Göttingen Nachrichten	German	1896	Laugel, Léonce	Sur une représentation géométrique d'un développement en fraction continue ordinaire	Translation
1896	Minkowski, Herman	Ueber Eigenschaften von ganzen Zahlen, die durch räumliche Anschauung erschlossen sind	Mathematical Papers Read at the International Mathematical Congress..., 1893	German	1896	Laugel, Léonce	Sur les propriétés des nombres entiers qui sont dérivées de l'intuition de l'espace	Translation
1896	Hurwitz, Adolph	Ueber die Reduction der binären quadratischen Formen	Mathematical papers read at the International Mathematical Congress..., 1893	German	1897	Laugel, Léonce	Sur la réduction des formes quadratiques binaires ; par M. A. Hurwitz	Translation
1897	Hurwitz, Adolph	Ueber lineare Formen mit ganzzahligen Variabeln	Göttinger Nachrichten	German	1989	Laugel, Léonce	Sur les formes arithmétiques linéaires à coefficients réels quelconques ; par M. A. Hurwitz	Translation

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TEXTS & DOCUMENTS

CHARLES HERMITE'S LETTERS TO FRANCISCO GOMES TEIXEIRA

PEDRO J. FREITAS

ABSTRACT. — It is well known that Charles Hermite kept an intense correspondence with many of the world's leading mathematicians of his time. This paper focuses on Hermite's letters to Francisco Gomes Teixeira, a Portuguese mathematician, who exchanged letters with Hermite for more than twenty years.

RÉSUMÉ (TEXTES & DOCUMENTS : Les lettres de Charles Hermite à Francisco Gomes Teixeira)

Il est bien connu que Charles Hermite a maintenu une correspondance intense avec nombre des plus grands mathématiciens de son temps. Cet article est consacré aux lettres d'Hermite à Francisco Gomes Teixeira, un mathématicien portugais ; leur échange a duré pendant plus de vingt ans.

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2000 Mathematics Subject Classification : 01A55, 01A70.

Key words and phrases : Charles Hermite, scientific correspondence, Francisco Gomes Teixeira.

Mots clefs. — Charles Hermite, correspondance scientifique, Francisco Gomes Teixeira.

1. INTRODUCTION

Francisco Gomes Teixeira (1851-1933) was one of the most important Portuguese mathematicians of the late nineteenth and early twentieth centuries. Having graduated from the University of Coimbra in 1875, he got his doctorate later in the same year, and, in the next year, started teaching at this university. A few years later, in 1884, he moved to the Polytechnic Academy of Porto, which would become the University of Porto in 1911. At this time, he became Rector of the newly established university.

In addition to his scientific works, mostly in analysis, published in both Portuguese and foreign journals, he innovated in the field of teaching, by writing college textbooks with ambitious content, which elevated the level of rigour of his time in Portugal. [1]

One may say that Gomes Teixeira's most relevant contribution for the evolution of mathematics in Portugal was the broadening of its scope, from the national to the international level. He received two prizes from the Royal Academy of Sciences of Madrid, in 1895 and 1897, which are representative of this interest in establishing contacts at an international level. The second of these prizes was awarded for a treatise on special curves, *Tratado de las curvas especiales notables, tanto planas como alabeadas*, which was later expanded, translated to French, and republished in 1917, after receiving another prize from the French Academy of Sciences. For more on his life and work, see for instance Vilhena [1936] and Alves [2004].

Along with the activity we have mentioned, there were several aspects which are central to this internationalization of Portuguese mathematics achieved by Gomes Teixeira. One of these was the foundation, in 1877, of the scientific journal *Jornal de Sciencias Matemáticas e Astronómicas*, which we will simply refer to as JSMA. For more on this journal, see Saraiva [2014] and Kharlamova [2013], a thesis that studies the importance of the journal in its time. [2]

The Journal benefited greatly from Gomes Teixeira's intense correspondence with some of the most renowned mathematicians of his time. The letters that Gomes Teixeira received are kept in the Archive of the University of Coimbra, which includes more than two thousand letters, indexed and catalogued by Gomes Teixeira himself. As a contribution to the anal-

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1 The textbooks were *Curso de Análise infinitesimal, cálculo diferencial*, Porto, Typ. Ocidental, 1887 and *Cálculo integral*, Porto: Typ. Occidental, 1889, edited and extended in later editions.

2 The volumes of this journal are now fully digitized and can be found at `www.fc.up.pt/fa/index.php?p=nav&f=html.fbib-Periodico-oa`.

ysis of this estate, this paper focuses on his correspondence with Charles Hermite, one of the most represented authors: there are 19 letters from Hermite (some of them photographically reproduced in Alves [2004] and Kharlamova [2013]) which we present here, and comment on for the first time.

2. THE CORRESPONDENCE

Charles Hermite (1822–1901) was known for being a very prolific correspondent. The paper Goldstein [2018] gives a general overview of Hermite's abundant correspondence (thousands of letters written). The paper also notes that since the beginning of the twentieth century, the letters Hermite sent to many mathematicians have been gradually edited—our present paper wishes to continue this trend.

Most of the correspondence received by Hermite was lost in a fire, which unfortunately makes it impossible to confront the letters in the Coimbra archive with the ones sent by Gomes Teixeira. The nineteen letters we present here go from 1872 to 1896, the number of letters for each year is as follows.

Year	72	75	79	81	85	86	88	90	91	92	96
Nb. of letters	1	2	1	2	3	2	2	2	1	2	1

We refer to the numbers of each letter in the catalogue (which can be found in Vilhena [1936]), assigned by Gomes Teixeira. However, our edition follows the chronological order, which does not correspond entirely to the order of the numeration in the catalogue.

Let us remark that there are nine other letters in Gomes Teixeira's archives, referring to the homage organized for Hermite on the occasion of his 70th birthday, in 1892—a subscription was made to present Hermite with a gold medal struck in his honor. Gomes Teixeira was invited by Gösta Mittag-Leffler to be a member of the honor committee for this event. The letters are from Mittag-Leffler, Gaston Darboux, Miguel Merino, Juan Jacobo Duran Loriga, Zoel Garcì de Galdeano, Lauro Clariana Ricart and other undisclosed senders, most of them asking for participation in the homage.

2.1. *July 17, 1872*

This is letter 141, and is the earliest letter from Hermite that can be found in this archive. Gomes Teixeira was still a university student at this point, finishing his 3rd academic year and starting the 4th. This letter, including a clear reference to the journal he would establish only five years later, reveals that, even as a 21 year-old student, Gomes Teixeira already planned to launch a new Portuguese mathematical journal and did not hesitate to write to one of the most famous mathematicians of his time asking for collaboration.

The paper contained in the letter and which we do not reproduce, is Hermite [1878], on the Frenet-Serret formulas for curves in 3-dimensional space. It was the first paper by a non-Portuguese author published in the JSMA. It can also be found in [Hermite 1917, pp. 508-511].

Monsieur,

Vous m'avez demandé, en commençant la publication de votre Journal des sciences mathématiques et astronomiques, de vous donner ma collaboration ; je viens remplir l'engagement que j'ai pris envers vous en vous adressant la note ci-jointe, concernant un point élémentaire de calcul différentiel.

Veuillez agréer Monsieur, l'expression de mes sentiments les plus distingués,

Ch. Hermite
Paris 17 Juillet 1872

2.2. *June 10, 1875*

This is letter 140 in the archive. In this letter, Hermite thanks Gomes Teixeira for sending him his inaugural dissertation (this is the name used for the Ph.D. thesis, see [Alves 2004, pp. 34ff.]), entitled *Integração das equações às derivadas parciais de 2a. ordem* (Integration of 2nd order partial differential equations). He takes the opportunity to send some of his own publications, without indication of their content.

Monsieur,

Je viens de parcourir la thèse inaugurale que vous m'avez fait l'honneur de m'adresser et quoique le sujet important et difficile que vous avez traité ne rentre point dans le cercle habituel de mes études, j'ai acquis l'assurance que vous avez fait un travail très sérieux et approfondi.

Veuillez accepter, Monsieur, les opuscules qui accompagnent cette lettre comme un témoignage de ma sympathie et recevoir l'expression de ma considération distinguée.

Ch. Hermite
Paris, 10 juin 1875

2.3. *December 3, 1875*

This is letter 765 in the archive, and contains an exercise given by Hermite to his students on the subject of continued fractions, one of Teixeira's interests. The exercise has some interest outside the context of student competitions: it provides a new proof of a result by Johann Lambert, quoted in the letter. Lambert is credited with the first proof that π is irrational (a proof which also uses continued fractions) in Lambert [1768]. This paper also studies powers of e, so this is probably the paper Hermite refers to in the letter.[3]

We note that the exercise in this letter, with its solution, does not appear among Hermite's contributions to the *Nouvelles annales de mathématiques*, nor is it published in Gomes Teixeira's JSMA.

Monsieur,

Je m'empresse de me conformer à vos intentions en vous accusant réception du mémoire que vous m'avez adressé. Le sujet que vous avez traité est d'une grande importance et touche à des questions dont je me suis moi-même occupé. C'est vous dire que je lirai votre travail avec beaucoup d'intérêt aussitôt que mes devoirs d'enseignement m'en donneront le loisir. J'aurai terminé au mois de Mars mon cours à l'Ecole polytechnique, et si d'ici là je n'ai pas pu trouver le temps d'étudier votre travail, vous pouvez compter qu'à cette époque du mois de Mars je m'en occuperai immédiatement.

Je saisis Monsieur cette occasion pour vous donner communication d'une petite question qui a été le sujet de la dernière composition donnée a mes élèves, et qui se rapporte précisément à la théorie des fractions continues. J'ai proposé de démontrer qu'en posant

$$F(x) = 1 + \frac{x}{1} + \frac{x^2}{1.2} + \cdots + \frac{x^n}{1 \cdot 2 \cdots n},$$

de sorte que

$$e^x = F(x) + \frac{x^{n+1}}{1 \cdot 2 \cdots (n+1)} + \text{etc.},$$

la dérivée d'ordre n de l'expression :

$$\frac{e^x - F(x)}{x^{n+1}}$$

a la forme suivante :

$$\frac{e^x \varphi(x) - \psi(x)}{x^{2n+1}},$$

[3] As is well-known, Hermite himself proved in 1873 that e is transcendental, Hermite [1873a], see also his allusions to Lambert in two letters he sent the same year to Paul Gordan and Carl Borchardt, Hermite [1873b;c].

où $\varphi(x)$ et $\psi(x)$ sont des polynômes, à coefficients entiers, du degré n. Vous voyez ainsi que $\frac{\psi(x)}{\varphi(x)}$ est l'une des réduites de la fraction continue qui représente l'exponentielle e^x, et cette circonstance que les coefficients de $\varphi(x)$ et $\psi(x)$ sont des nombres entiers conduit à une démonstration immediate de la proposition découverte par Lambert, que toutes les puissances de e sont incommensurables. Il suffit en effet d'observer que la dérivée d'ordre n de la série

$$\frac{1}{1\cdot 2\cdots(n+1)}+\frac{x}{1\cdot 2\cdots(n+2)}+\cdots$$

étant :

$$\begin{aligned}\frac{1}{(n+1)\cdot(n+2)\cdots(2n+1)}&+\frac{1}{(n+2)\cdot(n+3)\cdots(2n+2)}\,\frac{x}{1}\\&+\frac{1}{(n+3)(n+4)\cdots(2n+3)}\,\frac{x^2}{1\cdot 2}\\&+\frac{1}{(n+4)(n+5)\cdots(2n+4)}\,\frac{x^3}{1\cdot 2\cdot 3}+\cdots\end{aligned}$$

on a pour la quantité :

$$e^x\varphi(x)-\psi(x)$$

un développement en série qu'on peut écrire sous la forme suivante:

$$\frac{x^{2n+1}}{(n+1)(n+2)\cdots(2n+1)}\left[1+\frac{n+1}{2n+2}\,\frac{x}{1}+\frac{(n+1)(n+2)}{(2n+2)(2n+3)}\,\frac{x^2}{1\cdot 2}+\cdots\right].$$

On voit ainsi que le nombre n croissant, cette expression peut devenir sans jamais s'annuler, plus petite que toute quantité donnée; car le facteur $\frac{x^{2n+1}}{(n+1)(n+2)\cdots(2n+1)}$ a pour limite, zéro, tandis que la série entre crochets est toujours finie, les termes étant respectivement moindres que ceux de $e^x=1+\frac{x}{1}+\cdots$

Veuillez Monsieur m'excuser du retard que mes occupations m'obligent de mettre à la lecture de votre travail et recevoir la nouvelle assurance de ma considération distinguée

Ch. Hermite
Paris, 2 rue de la Sorbonne, 3 décembre 1875

2.4. *November 19, 1879*

This is letter 142 in the archive, and contains the paper Hermite [1880], which can also be found in [Hermite 1917, pp. 505–507]. It is a result about an integral involving trigonometric functions, which Hermite proved in his *Cours d'Analyse.* The paper contains a simpler proof of the result. The letter also contains some results about interpolating polynomials.

Paris, 19 novembre 1879
(2 rue de la Sorbonne)

Monsieur,

Ce m'est un grand regret que sur le petit nombre de 25 exemplaires du tirage à part de mon article sur l'article [*sic*] concernant l'interpolation, il ne m'en reste plus un seul que je puisse vous offrir pour répondre à votre désir. Permettez moi de me dédommager un tant soit peu en vous envoyant avec cette lettre toutes celles de mes publications dont je puis disposer et aussi en vous donnant en quelques mots le principe de ma formule. La question étant de déterminer un polynôme $F(x)$ de degré $n-1$ satisfaisant aux conditions :

$$\begin{array}{llll} F(a) = f(a), & F(b) = f(b), & \ldots, & F(l) = f(l), \\ F'(a) = f'(a), & F'(b) = f'(b), & \ldots, & F'(l) = f'(l), \\ \vdots & \vdots & & \vdots \\ F^{\alpha-1}(a) = f^{\alpha-1}(a), & cF^{\beta-1}(b) = f^{\beta-1}(b), & \ldots, & F^{\lambda-1}(l) = f^{\lambda-1}(l), \end{array}$$

où $f(x)$ est une fonction donnée; en supposant $\alpha + \beta + \cdots + \lambda = n$, je considère une aire S, comprenant d'une part, $a, b, \ldots, l$, et de l'autre la quantité variable x, j'admets qu'à son intérieur la fonction $f(x)$ soit uniforme et n'ait aucun pôle; cela étant, on a :

$$F(x) - f(x) = \frac{1}{2i\pi} \int \frac{f(z) \cdot (x-a)^\alpha (x-b)^\beta \cdots (x-l)^\lambda}{(x-z).(z-a)^\alpha (z-b)^\beta \cdots (z-l)^\lambda} dz,$$

l'intégrale du second membre se rapportant au contour de S. L'utilité principale de cette relation est moins de déterminer le polynôme cherché $F(x)$ par le calcul de l'intégrale, et au moyen d'une somme de résidus, que de montrer que la différence $F(x) - f(x)$, diminue sans limite, lorsque le nombre des quantités, $a, b, \ldots, l$, ou bien les exposants $\alpha, \beta, \ldots, \lambda$ vont en augmentant.

À ma bien légère offrande d'une note pour votre Journal, je joins Monsieur l'expression de ma considération la plus distinguée et de mes sentiments dévoués

Ch. Hermite

2.5. *July 16, 1881*

This is letter 143 in the archive. In this letter, Hermite comments on a note Gomes Teixeira had sent him to be presented at the Académie des Sciences, directing him to previous work on the topic. About this letter, Gomes Teixeira writes in the catalogue (see Vilhena [1936]) that he later sent a new result which was indeed published in the *Comptes-rendus* of the French Academy (namely Gomes Teixeira [1881]). The topic of this second note is the integration of a partial differential equation, the same topic as his thesis. Letter 764 (immediately below) acknowledges the reception of this second paper, and contains some more comments.

It is remarkable that, after gracefully declining the publication of the first note, Hermite encourages Gomes Teixeira to send a new one, acknowledging him as a representative of mathematics both for Portugal and Spain.

Paris, 16 juillet 1881

Monsieur,

Le travail que vous m'avez communiqué se rapportant à des questions d'analyse dont Mr Darboux s'est occupé spécialement et avec le plus grand succès, je l'ai prié de vouloir bien en prendre connaissance et c'est son avis autant que le mien dont je viens vous faire part.

Nous croyons qu'il vous serait nécessaire d'étudier plusieurs recherches récentes et importantes dont vous ne paraissez pas avoir eu connaissance, notamment celles de Bour [4] et aussi les mémoires d'Imenetcky et de Mr Pelet. [5]

Votre nom représentant les sciences mathématiques pour votre pays et aussi pour l'Espagne, nous croyons devoir vous conseiller de ne le produire pour la première fois dans les Comptes-Rendus qu'a l'occasion d'un travail complètement digne de cette situation. Votre talent Monsieur, et je suis heureux de vous le dire au nom de Mr Darboux comme au mien, vous appelle à le remplir de la manière la plus honorable. Nous pensons mieux vous témoigner notre sympathie en vous demandant de nouveaux efforts qu'en publiant un travail dont on pourrait dire qu'il n'a pas été accueilli sans quelque indulgence, et c'est en espérant que nous aurons bientôt les fruits de ces efforts que je vous offre Monsieur la nouvelle expression de toute mon estime et de mes sentiments bien dévoués,

Ch. Hermite

2.6. *November 1, 1881*

This is letter 764 in the archive.

Monsieur,

Je m'empresse de vous accuser réception de la note que vous m'avez adressée et de vous informer que je remplirai vos intentions avec grand plaisir en la présentant a la prochaine séance de l'Académie, afin qu'elle soit publiée

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4 Edmond Bour (1832–1866) studied both at the École Polytéchnique and the École des Mines. He published articles on differential equations, celestial mechanics and the geometry of surfaces.

5 Respectively Vasilij Grigorjevič Imšenecki (1832–1892) and Auguste Pellet (1848–1935).

dans les Comptes-Rendus. J'espère que plutard[6] vous donnerez dans votre Journal des applications qui en feront ressortir l'utilité et l'importance.

Veuillez agréer Monsieur la nouvelle assurance de mes sentiments bien dévoués,

Ch. Hermite
Paris 1er novembre 1881

2.7. *May 31, 1885*

This is letter 766 in the archive and contains the paper Hermite [1885], which also appears in [Hermite 1917, pp. 169–171]. It concerns a relation between Legendre polynomials and continued fractions of functions (we include only the beginning of what is reproduced verbatim in the published paper[7]).

Paris, 31 mai 1885

Monsieur,

Je viens vous remercier des explications que vous avez eu la bonté de me donner et qui m'ont permis de mieux saisir votre analyse, à laquelle je n'ai plus d'objections à faire. Des applications de la formule à laquelle vous êtes parvenu, à des cas particuliers simples me sembleraient utiles pour en faire comprendre le caractère; mais en ce moment je ne puis même songer a vous donner des indications précises comme il serait nécessaire, les devoirs dont je suis chargé ne me laissant aucune liberté d'esprit.

Je voudrais cependant Monsieur vous offrir un témoignage de mes sentiments de sympathie et d'estime en vous donnant pour être publiée dans votre Journal, si vous le voulez bien, une petite remarque qui a fait le sujet d'une de mes dernières leçons à la Sorbonne.

[Here begins the reproduction in Gomes Teixeira's JSMA:

"Vous connaissez cette belle proposition de M. Tchebychew que le polynôme X_n de Legendre est le dénominateur de la réduite d'ordre n du développement en fraction continue de la quantité :

$$\frac{1}{2}\log\frac{x+1}{x-1} = \frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \cdots$$

On peut y parvenir comme vous allez voir, au moyen du développement en série qui a été le point de départ de Legendre et a donné la première des for-

6 That is, "later", an incorrect version of "plus tard." This word is not currently in use, but it does appear in other nineteenth-century sources, e.g., in a letter from Gustave Flaubert to George Sand, dated July 27, 1871—see Larthomas [1999].

7 A misprint for the sign used in the expression of ξ, [Hermite 1885, 83], was corrected in Hermite's complete works.

mules des polynômes X_n, à savoir :

$$\frac{1}{\sqrt{1-2zx+z^2}} = X_0 + X_1 z + \cdots + X_n z^n + \cdots .$$

.... Remarquez encore que

$$F(0) = \sum \frac{X_i X_{n-i-1}}{i+1}$$

est un polynome entier en x du degré $n-1$; $\frac{F(0)}{X_n}$ est bien par conséquent la réduite d'ordre n du développement de $\frac{1}{2}\log\frac{x+1}{x-1}$ en fraction continue."]

Veuillez Monsieur recevoir la nouvelle assurance de toute ma sympathie et de mes sentiments bien dévoués

Ch. Hermite

2.8. *October 20, 1885*

This is letter 767 in the archive. The paper mentioned in this letter is Gomes Teixeira [1885], which proves that if a power series $y = \sum a_n x^n$ with rational coefficients satisfies a differential equation $P(x, y, y') = 0$, for a polynomial P with integer coefficients, then the prime factors of the denominators of the a_n do not increase too much. [8] The following letters, in particular that of January 26, 1886 (number 770, below), also refer to the same topic.

Monsieur,

J'espère n'avoir pas été contre vos intentions en demandant à mon confrère Mr Darboux de publier dans les Annales de l'École Normale Supérieure le travail que vous avez fait l'honneur de me communiquer. La proposition que vous avez ajoutée aux résultats importants découverts par Eisenstein sur les séries à coefficients rationnels qui satisfont à une équation algébrique a coefficients entiers nous a tous deux beaucoup intéressés, et je suis heureux Monsieur de joindre mes félicitations à l'assurance de ma haute estime et à celle de mes sentiments bien dévoués.

Ch. Hermite
20 octobre 1885

2.9. *December 14, 1885*

This is letter 769 in the archive. In this letter there is no reference to any paper. Apparently, Gomes Teixeira sent a theorem to Hermite (possibly in a previous letter), without proof, and Hermite encourages him to find one,

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8 The Eisenstein statement alluded to by Hermite deals with an analogous result in the situation $P(x, y) = 0$, Eisenstein [1852].

taking the opportunity to make interesting remarks about the need for the presentation of mathematical results to be as simple and clear as possible, a constant concern in these letters.

Paris 14 décembre 1885

Monsieur,

L'énoncé du théorème que vous m'avez fait l'honneur de me communiquer me fait voir que vous êtes engagé dans une voie excellente, et je ne puis que vous engager vivement à faire tous les efforts nécessaires pour arriver à une démonstration simple et facile qui d'elle-même devienne classique. Vous le savez Monsieur, et je n'ai pas à vous l'apprendre qu'après le travail de l'invention, il en est un autre dont nos maîtres en Analyse ont donné l'exemple et le modèle, de sorte que les œuvres de Gauss et de Jacobi joignent à l'importance à l'éclat des découvertes, le mérite d'une forme parfaite. En vous exprimant l'espoir qu'ayant été assez heureux pour obtenir un beau résultat, vous réussirez aussi [à] l'exposer sous la forme la meilleure, je saisis Monsieur cette occasion pour vous renouveler l'expression de tous mes sentiments de sympathie et d'estime.

Ch. Hermite.

2.10. *January 26, 1886*

This is letter 770 in the archive. It refers to paper Gomes Teixeira [1886], with a special note to the elegance and simplicity of the proof provided.

Monsieur

J'ai donné communication à Mr Darboux de la démonstration du théorème d'Eisenstein que vous m'avez fait l'honneur de m'écrire, et nous pensons remplir vos intentions en la publiant dans les Annales de l'École Normale, à la suite du premier article que vous avez donné a ce recueil.

En vous félicitant, Monsieur de la simplicité et de l'élégance de votre démonstration, je vous prie d'agréer l'assurance de toute mon estime et de mes sentiments dévoués,

Ch. Hermite
Paris 26 janvier 1886

2.11. *February 12, 1886*

This is letter 768 in the archive. The article this letter refers to is Gomes Teixeira [1887], which is a development of the result in paper Gomes Teixeira [1885], published before, also in the *Annales de l'École Normale Supérieure.* After publication, Adolf Hurwitz remarked that Gomes Teixeira's theorem, as stated in 1887, is incorrect and presented a counter-example; he also suggested a new, more intricate, statement for the theorem and proved it in Hurwitz [1889].

Monsieur,

J'ai le plaisir de vous informer que la dernière communication que vous avez fait l'honneur de m'adresser sera publiée, comme les précédentes, dans les Annales de l'École Normale.

En vous renouvelant Monsieur mes sincères félicitations pour ce nouveau résultat de votre travail, je vous prie d'agréer l'assurance de mes sentiments les plus distingués.

Ch. Hermite
Paris 12 février 1886

2.12. *November 16, 1888*

This is letter 772 in the archive. It mentions two letters sent by Gomes Teixeira. In the present letter, Hermite provides a new proof of a formula sent in the second of Gomes Teixeira's letters. He also offers publication of a paper, probably Gomes Teixeira [1888a] (it may have been sent with the first letter).

There are two figures illustrating the text, we include the original ones along with a reproduction.

16 novembre 1888

En lisant avec attention la seconde des deux lettres que vous m'avez fait l'honneur de m'adresser et dans laquelle vous parvenez a cette relation :

$$\int_a^b \varphi^2(x)dx \cdot \int_a^b \psi^2(x)dx - \left[\int_a^b \varphi(x)\psi(x)dx\right]^2 = \int_a^b dx \int_x^b [\varphi(x)\psi(y) - \varphi(y)\psi(x)]^2 dy,$$

ou bien, sous une forme un peu plus générale :

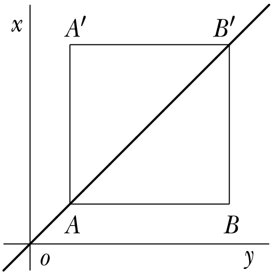
$$\int_a^b \varphi(x)\varphi_1(x)dx \cdot \int_a^b \psi(x)\psi_1(x)dx - \int_a^b \varphi(x)\psi_1(x)dx \cdot \int_a^b \varphi_1(x)\psi(x)dx$$
$$= \int_a^b dx \int_x^b [\varphi(x)\psi(y) - \varphi(y)\psi(x)][\varphi_1(x)\psi_1(y) - \varphi_1(y)\psi_1(x)]dy,$$

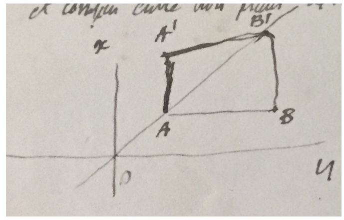
j'ai reconnu que cette relation est identique comme vous allez le voir.

Considérons en effet l'intégrale double :

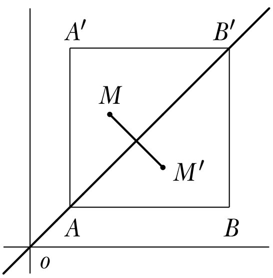
$$J = \int_a^x dx \int_x^b F(x,y)dy.$$

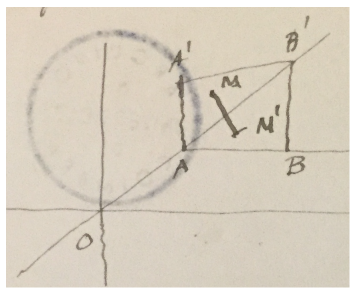
Elle représente le volume d'une sorte de prisme, limité par la surface $z = F(x, y)$ et compris entre trois plans déterminés comme il suit.





Tracez sur le plan des xoy la bissectrice $y = x$ de l'angle xoy puis le rectangle $ABB'A'$ dont elle serait la diagonale, et supposons que les abcisses [*sic*] des sommets A et B, de ce rectangle soient a et b. Les plans dont je parle, perpendiculaires au plan des xy auront pour traces les droites, AA', $A'B'$, AB' et la base du prisme considéré sur le triangle rectangle $AA'B$.





Soit M un point de l'intérieur de ce triangle, ayant pour coordonnées x et y ; son symétrique M' par rapport a AB' aura évidemment pour coordonnées y et x, par conséquent les valeurs de l'ordonnée $z = F(x, y)$, seront les mêmes en ces deux points lorsque la fonction $F(x, y)$ est symétrique en x et y. C'est le cas de l'expression qui entre dans votre intégrale double; on peut par conséquent effectuer l'intégration en prenant pour limite le rectangle au lieu du triangle, et divisant par deux; mais on obtient ainsi la quantité

$$J = \frac{1}{2}\int_a^b dx \int_a^b [\varphi(x)\psi(y) - \varphi(y)\psi(x)]^2 dy,$$

qui se ramène d'elle même à des intégrales simples.

Les considérations que vous exposez dans votre lettre du 23 Octobre m'ont beaucoup intéressé et me semblent excellentes ; afin de remplir vos intentions je demanderai à mon confrère M^{r} Darboux d'en prendre connaissance et s'il partage mon sentiment comme j'ai tout lieu de le penser, je le prierai de publier votre lettre soit dans les Annales de l'École Normale soit dans le Bulletin des Sciences Mathématiques.

Veuillez agréer Monsieur, la nouvelle assurance de mes sentiments de bien sincère sympathie et de haute estime

Ch. Hermite
Paris 16 novembre 1888

2.13. *November 22, 1888*

This is letter 773 in the archive. This letter answers a letter from Gomes Teixeira establishing an inequality which can be used to provide an upper bound for integrals of the type

$$\int_a^b \varphi(x)\psi(x)\,dx.$$

In his letter, Teixeira compared this result with an inequality by Pafnuty Chebyschev, which provides a lower bound for integrals of the same type. Hermite suggests a small correction and offers publication; this will be Gomes Teixeira [1888b]. The letter also refers to a previous one, sent by Gomes Teixeira to Hermite, which was published as Gomes Teixeira [1888a].

Paris, 22 novembre 1888

Monsieur,

Vous êtes parvenu à des conséquences qui m'ont paru très intéressantes en rapprochant du théorème de M. Tchebichew, votre relation

$$\int_a^b \varphi^2(x)\,dx \cdot \int_a^b \psi^2(x)\,dx > \left[\int_a^b \varphi(x)\psi(x)\,dx\right]^2.$$

Je m'en suis entretenu avec M$^{\text{r}}$ Darboux, en lui donnant communication de votre dernière lettre où vous les exposez avec plusieurs applications, et je ne pense point qu'il vous sera désagréable que cette lettre soit publiée dans le Bulletin des Sciences Mathématiques, comme celle que vous m'aviez déjà fait l'honneur de m'adresser. Les épreuves vous seront envoyés à Porto pour que vous puissiez les revoir et les corriger; vous me permettrez de vous proposer de remplacer votre intégrale double

$$\int_a^b dx \int_x^b [\varphi(x)\psi(y) - \varphi(y)\psi(x)]^2 dy$$

par celle-ci

$$\frac{1}{2}\int_a^b dx \int_a^b [\varphi(x)\psi(y) - \varphi(y)\psi(x)]^2 dy$$

afin qu'on reconnaisse immédiatement que vous partez, comme vous le dites, d'une identité.

En vous renouvelant, Monsieur, l'assurance de ma haute estime et de mes sentiments dévoués,

Ch. Hermite

2.14. *March 15, 1890*

This is letter 771 in the archive, and it refers to the paper Gomes Teixeira [1889], which provides a new proof of a result in Hermite's *Cours d'Analyse*, further developed in one of Hermite's articles in the JSMA, namely Hermite [1880]. Hermite compliments this proof, stating he will actually use it in one of his lessons, and gives yet another proof of the result.

Monsieur,

J'ai lu avec le plus grand plaisir dans le dernier n° du Journal des sciences mathématiques et astronomiques, la belle et savante méthode que vous avez exposée pour obtenir l'intégrale

$$\int_0^\pi \cot(x-a-ib)\,dx,$$

en la rattachant à l'expression générale

$$\int_0^\pi \frac{f'(x)\,dx}{1+f^2(x)},$$

et à la notion de l'indice de Cauchy.

Vous avez ainsi obtenu une application très intéressante de la théorie du grand géomètre, et je me propose de la mettre à profit dans une de mes leçons. A cette occasion, et pour vos élèves, permettez moi de vous indiquer un autre procédé qui conduit à la valeur de cette intégrale au moyen de la relation trigonométrique

$$\cot nx = \sum \frac{1}{n}\cot\left(x+\frac{k\pi}{n}\right), \qquad (k=0,1,2,\ldots,n-1).$$

À cet effet, je change d'abord x en $x-a-ib$, puis je fais croître indéfiniment l'entier n. En posant $\frac{\pi}{n}=dx$, vous voyez que le second membre a ainsi pour limite l'intégrale définie qu'il s'agit d'obtenir :

$$\frac{1}{\pi}\int_0^\pi \cot(x-a-ib)\,dx,$$

de sorte que l'on a pour n infiniment grand :

$$\int_0^\pi \cot(x-a-ib)\,dx = \pi\cot n(x-a-ib).$$

Cela étant, la formule :

$$\cot n(x-a-ib) = \frac{1}{i}\,\frac{e^{2in(x-a-ib)}+1}{e^{2in(x-a-ib)}-1}$$

donne immédiatement dans cette hypothèse, $+i$ ou $-i$ suivant que b est positif ou négatif, et par conséquent le résultat cherché.

Peut-être pensez vous comme moi Monsieur qu'il est bon dans l'enseignement de donner sous plusieurs points de vue la démonstration des propositions importantes, c'est le but que j'ai eu en vue. Je remarquerai encore que la relation dont j'ai fait usage :

$$\cot nx = \sum \frac{1}{n} \cot\left(x + \frac{k\pi}{n}\right),$$

résulte immédiatement de la décomposition en éléments simples de la fonction $\cot nx$, les résidus pour $x = -\frac{k\pi}{n}$, c'est à dire les valeurs que prend dans cette supposition le produit

$$\left(x + \frac{k\pi}{n}\right) \cot nx = \frac{\left(x + \frac{k\pi}{n}\right) \cos nx}{\sin nx},$$

étant tous égaux a $\frac{1}{n}$.

Une dernière remarque; l'intégrale $J = \int_0^\pi \cot(t - z) dt$, considérée comme fonction de la variable z, offre une coupure représentée par l'équation, $t - z = k\pi$, où t varie de zéro à π, k étant un entier arbitraire, cette coupure est donc l'axe des abscisses. Dans le demi plan au dessus et dans le demi plan au dessous de cet axe, l'intégrale est une fonction continue uniforme de z et la relation $D_z J = 0$ qu'on obtient immédiatement montre qu'elle est constante. Pour l'obtenir je suppose $z = x + iy$, et en attribuant à y successivement des valeurs infiniment grandes, positives et négatives, je trouve encore :

$$J = \int_0^\pi i dt, \qquad J = -\int_0^\pi i dt,$$

c'est-à-dire, $J = i\pi$ et $J = -i\pi$.

Veuillez agréer Monsieur, la nouvelle assurance de mes sentiments bien dévoués

Ch. Hermite

J'abrège un peu le calcul que vous avez donné, en partant de l'identité

$$\cot(x - y) - \cot(x + y) = \frac{2 \sin y \cos y}{\sin^2 x - \sin^2 y},$$

d'où l'on tire :

$$\int [\cot(x - y) - \cot(x + y)] dx = \int \frac{2 \sin y \cos y \, dx}{\sin^2 x - \sin^2 y}$$

Puis si l'on pose $z = \tan y \tan x$

$$\int [\cot(x - y) - \cot(x + y)] dx = \int \frac{dz}{1 - z^2}.$$

Il suffit ensuite de changer x en $x - a$ et de faire $y = ib$.

Paris 15 mars 1890

2.15. *July 9, 1890*

This is letter 774 in the archive. The letter refers to the article Gomes Teixeira [1890], where a complex integral is studied in order to provide results about series of trigonometric functions. There is also a reference to a text sent to Hermite by José Pedro Teixeira (1857–1925), a professor at the Polytechnic Academy of Porto.

Paris 9 juillet 1890

Monsieur,

Le travail dont vous avez bien voulu me donner communication m'a semblé excellent, et j'espère que je n'aurai pas été contre vos intentions en le faisant lire à M^r Darboux qui m'a exprimé le désir de le publier dans le Bulletin des sciences mathématiques. J'ose penser que vous donnerez votre consentement à cette publication qui est un témoignage de nos sentiments de sympathie à votre égard et de notre estime pour votre talent. Vous voudrez bien aussi Monsieur m'excuser de ne vous soumettre aucune remarque sur votre analyse, les devoirs universitaires sont tellement onéreux à ce moment, à cause du grand nombre d'examens dont je suis chargé à la Sorbonne, que le courage et le temps me manquent pour pouvoir m'occuper de sujets élevés de calcul intégral.

En saisissant cette occasion pour remercier M^r José Pedro Teixeira du don de son mémoire sur les fonctions elliptiques, et en vous exprimant toute ma satisfaction de voir cette théorie cultivée avec autant de succès, je vous prie Monsieur de recevoir la nouvelle assurance de ma plus haute estime et de mes sentiments bien dévoués.

Ch. Hermite

2.16. *November 9, 1891*

This is letter 775 in the archive. The letter refers to the article Gomes Teixeira [1892], which is a study of $\wp(u)$, Karl Weierstrass's elliptic function.

Paris 9 novembre 1891

Monsieur,

C'est avec le plus grand plaisir que j'ai lu l'analyse si simple et si élégante dont vous m'avez donné communication. Votre méthode me semble ouvrir une des voies les meilleures pour entrer dans la théorie des fonctions elliptiques, et dans le désir que l'on profite du fruit de vos efforts, j'ai pensé que vous voudriez bien consentir à la publication de votre lettre dans le Bulletin des sciences mathématiques. Je me fais un agréable devoir de vous exprimer le sentiment de mon confrère M^r Darboux qui apprécie comme moi votre beau talent, et je saisis Monsieur cette occasion pour vous renouveler l'expression de ma haute estime et de ma bien sincère sympathie

Ch. Hermite.

2.17. *May 24, 1892*

This is letter 776 in the archive. In it, Hermite thanks Gomes Teixeira for having sent his *Curso de análise*, and makes very specific remarks about it, complimenting the clarity and elegance of the text, which denotes a careful reading. Hermite also promises to send a note for publication: it will be done only later (see the following letter, 777) and on a topic different from the one announced.

Paris 24 mai 1892

Monsieur,

Les trois volumes de votre Cours d'Analyse infinitésimale que vous m'avez envoyés pour être offerts à l'Académie des sciences, lui ont été présentés avec éloges par M^{r} le Secrétaire perpétuel dans la séance de lundi dernier et vous verrez la mention de cette présentation dans le Compte-rendu de cette séance. Vous ne pouvez douter Monsieur du grand intérêt avec lequel j'ai lu la seconde partie de votre Calcul intégral, et tout particulièrement la théorie des fonctions doublement périodiques et des fonctions elliptiques. Il me parait difficile d'atteindre dans l'exposition un plus grand degré de simplicité, de clarté et d'élégance, et je ne puis douter que tous les amis de la science, les géomètres comme les commençants ne vous rendent le même témoignage. J'aurais bien désiré vous donner autrement qu'en paroles une preuve de mes sentiments de bien grande estime, en vous envoyant un article pour le Journal que vous dirigez avec tant de succès et j'avais songé dans ce but à rédiger une recherche sur les nombres de Bernouilli [*sic*] qui a fait le sujet d'une de mes leçons. Mais je suis dans la nécessité d'ajourner ce travail, étant à cette époque de l'année surchargé d'ouvrage ; il me faut attendre un moment où j'aurai plus de liberté, et vous voudrez bien me faire crédit d'un peu de temps.

En vous demandant de prendre acte de mon engagement, je saisis Monsieur cette occasion pour vous renouveler l'assurance de ma plus haute estime, et celle de mes sentiments bien sincèrement dévoués

Ch. Hermite

2.18. *December 2 and 4, 1892*

This is letter 777; it mentions two dates, one at the beginning and one at the end. The archived material does not include the note promised in this letter. As it happens, there is a small article by Hermite in the JSMA, volume 11, namely Hermite [1892] (it can also be found in [Hermite 1917, pp. 512–513]). As there are no other publications by Hermite in the JSMA, other than the three identified above and this one, we believe this note to be the one mentioned in this letter. It concerns a property of elliptic functions and elliptic integrals, providing a proof for a formula about addition of arguments in Weierstrass's elliptic function $\wp(x)$.

Paris 4 décembre 1892

Monsieur,

J'ai un double remerciement à vous adresser; je vous rends grâces d'abord pour votre communication dont j'ai donné connaissance, en pensant remplir votre intention, à mon confrère de l'Académie des sciences, M^r Darboux, qui l'a accueillie avec empressement pour la publier dans le Bulletin des sciences mathématiques. Il me faut ensuite vous témoigner ma vive gratitude d'avoir bien voulu joindre votre nom estimé de tous les géomètres à ceux de mes amis mathématiques qui ont désiré m'offrir un témoignage de sympathie à l'occasion de mes 70 ans. La sympathie est réciproque, et vous y avez un droit particulier, en raison du talent et du zèle que vous consacrez à votre journal des sciences mathématiques et astronomiques, et du service éminent que vous rendez ainsi à la science de votre pays. Peut-être n'avez vous pas oublié que je vous avais promis une note que devrait concerner le théorème de Staudt sur les nombres de Bernouilli [*sic*], mais mon travail a pris une autre direction et c'est avec un autre sujet que je m'acquitterai envers vous. Permettez moi Monsieur de vous la transcrire ci-après et de saisir cette occasion pour vous renouveler l'assurance de ma haute estime et de mes sentiments bien dévoués

Ch. Hermite
Paris 2 décembre 1892

2.19. *January 28, 1896*

This is letter 30 in the archive. It contains a request from Hermite to add Gomes Teixeira's name to a letter in support of *Acta Mathematica*, the journal founded in 1882 by Gösta Mittag-Leffler, which was going through difficult financial problems at the time. The request, says Hermite, is so urgent that an absence of response will be considered as an affirmation of support. It is clearly a text that was sent to many scientists at the time: a similar letter can be found in Ogigova [1967], with the same date, this time addressed to Andrei Markov; this article also transcribes the text of the circular letter from Nörlund [1927], where the names of 30 signatories, among them Gomes Teixeira, can be found. It adds that there are 370 more signatures, for a total of 400 mathematicians.

Monsieur,

Permettez moi de faire un appel à votre bonne obligeance en sollicitant votre concours en faveur d'un intérêt scientifique au quel j'espère vous ne refuserez pas votre sympathie.

Les Acta Mathematica traversent en ce moment une crise qui met leur existence en grand péril; la commission du budget de la diète suédoise a manifesté l'intention de supprimer entièrement l'allocation, après l'avoir diminuée de moitié l'année dernière, qu'elle avait accordée au Journal, depuis sa fondation. Ce n'est pas seulement en France que les économies sont recherchées et réclamées sans pitié, et qu'une assemblée politique se soucie peu des géomètres. M^r Mittag-Leffler et les Acta ne sont point cependant sans défenseurs, par une circonstance heureuse l'un des plus dévoués est le président même de la commission du budget à la diète. Ce haut patronage a émis l'avis qu'une manifestation venue de l'étranger, apportant au nom de savants autorisés un témoignage public de l'importance des Acta et du mérite élevé de son fondateur, déciderait certainement un vote favorable de l'assemblée. L'Académie des sciences de Paris est entrée dans ces vues avec empressement et à l'unanimité; elle a pris l'initiative d'une adresse dont le 50^e anniversaire de la naissance de M^r Mittag-Leffler offrait l'occasion, pour lui exprimer ses sentiments de sympathie, en lui faisant don de son portrait. En même temps, elle se propose d'adresser une circulaire aux géomètres pour obtenir leur adhésion, et j'ai reçu Monsieur la mission de vous soumettre la circulaire et l'adresse, en vous demandant de permettre que votre nom soit joint à ceux de mes confrères de l'Académie des sciences et des mathématiciens qui nous ont déjà donné leurs concours. Comme le temps nous est strictement mesuré, je prends la liberté de vous demander une réponse, pour le seul cas où elle ne répondrait pas à notre attente, et par une simple carte postale, dans le moindre délai qui vous sera possible, en considérant que votre acceptation nous est acquise si je ne reçois pas d'avis contraire.

Dans l'espérance que je n'aurai pas fait appel en vain à votre concours, dans cette circonstance, je saisis Monsieur cette occasion pour renouveler l'assurance de ma plus haute estime et de mes sentiments bien dévoués.

Ch. Hermite
Paris 28 janvier 1896

3. CONCLUSIONS

As we consider these texts, we would like to draw attention to two aspects which become apparent in the relationship between Hermite and Gomes Teixeira.

This correspondence shows a notable personality trait of Hermite: his desire to continuously support and encourage a younger colleague, in a country that did not have a very strong international presence in mathematics. Even if we take into account the courteous formalities in the text, one can nevertheless say that there is some genuine interest and encouragement from Hermite's part towards Gomes Teixeira.

It is likewise remarkable that Gomes Teixeira started this correspondence at a very early age: he was still 21 years old when he first contacted Hermite, one of the leading names in mathematics, asking him to participate in a journal that was, at the time, only a project. It is equally remarkable that Hermite did send a paper in reply.

The texts of the letters presented here may not show the same depth as the ones between Hermite and others of his numerous correspondents. To take only two examples, the correspondence with Mittag-Leffler (which can be found at the *Cahiers du séminaire d'histoire des mathématiques*, see Hermite [1984], Hermite [1985] and Hermite [1989]) denotes a more personal relationship, and the letters to Jacobi (some of which can be found at the *Journal für die reine und angewandte Mathematik*, see Hermite [1850]) show greater mathematical depth. However, this correspondence was maintained throughout 25 years, from 1872 to 1896, and tells of a persistent relationship between the two mathematicians. Moreover, the nine letters addressed to Gomes Teixeira about the homage to Hermite in 1892 (which we have not not included here) show that this proximity was acknowledged by the mathematical community of their time.

This analysis of the correspondence from Hermite to Gomes Teixeira is, to the best of our knowledge, the first in-depth study of the material in the archive of the correspondence received by Gomes Teixeira. It focuses only on 19 letters, out of the more that 2,000 that are extant in this archive. We hope to continue this work in following publications, focusing on other aspects of Gomes Teixeira's correspondence.

4. ACKNOWLEDGEMENTS

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Edited under the auspices of the French Mathematical Society (Société Mathématique de France), the Journal for the History of Mathematics *publishes original papers (in French or in English) devoted to the history of mathematics, from Antiquity to the present. The* Journal *welcomes manuscripts dealing with the development of the mathematical sciences proper as well as papers bearing on relationships to other disciplines or on the institutional, cultural, and social contexts. The ambition of the* Journal for the History of Mathematics *is to serve the historians of mathematics' international community by offering a forum for critical debate, open to historiographic essays and programmatic contributions. Beyond the professional community, the* Journal *is addressed to mathematicians, historians and philosophers of science, sociologists, anthropologists, and to all those interested in understanding mathematics and its development.*

À propos des indicateurs bibliographiques / *Bibliographic indicators*

Le Comité de rédaction de la *Revue d'histoire des mathématiques* souhaite exprimer sa position sur les facteurs d'impact et autres indicateurs présents sur le marché des revues scientifiques, comme le taux d'acceptation des articles. En 2009, la *Revue d'histoire des mathématiques* avait déjà, comme la très grande majorité des revues d'histoire des sciences, signé l'appel « Journals under Threat : A Joint Response from HSTM Editors », contre le classement en A, B, C de ces revues. Tant les mathématiciens que les spécialistes de sciences humaines et sociales ont établi que les indicateurs bibliométriques usuels n'ont pas de pertinence individuelle — en particulier parce qu'ils varient d'une discipline à une autre, et même d'une sous-discipline à une autre —, qu'ils ne permettent pas d'évaluer la qualité scientifique d'articles ou d'auteurs, et que la plupart d'entre eux sont faciles à manipuler. Dans un domaine en plein développement théorique comme l'histoire des mathématiques, le Comité de rédaction estime aussi que la qualité d'une revue n'est pas mesurée par son refus d'une grande quantité d'articles (ce qu'il est toutefois amené à faire), mais par sa capacité à améliorer les articles par des rapports détaillés et par l'encadrement des auteurs jusqu'à la publication. Il ne rendra donc public aucun indicateur de ce type.

The Editorial Board of the Revue d'histoire des mathématiques *wishes to express its point of view concerning impact factors—and other indicators such as acceptance rates—that are currently being used in the scientific journal market. In 2009, the* Revue d'histoire des mathématiques*, like most history of science journals, signed the call "Journals under Threat: A Joint Response from HSTM Editors" against the ranking of these journals on an A, B, and C scale. Mathematicians as well as scholars in the social sciences and in the humanities have established that standard bibliometric indicators are meaningless for ranking individual papers; they vary from one discipline to another, and even from one sub-discipline to another, they also do not assess the scientific quality of articles and authors, and most are easy to tamper with. In a field in full conceptual development such as the history of mathematics, the Editorial Board also believes that the quality of a journal is not measured by its rejection of a large number of articles (which it is always obliged to do), but by its ability to improve articles through detailed referee reports and through working with authors at each step of the publication process. The Editorial Board of the* Revue d'histoire des mathématiques *will thus not make public any indicators of this kind.*

Sommaire

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